

# Performances of Classic and Weighted Versions of Some Selected Methods in Estimation of Spectral Data from Camera Responses

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**Abstract**—The reconstruction of spectral reflectances of different color charts from the corresponding RGB values is investigated. The pseudo inverse (PI), principal component analysis (PCA) and canonical correlation regression (CCR) as well as their weighted versions are employed to estimate the spectral reflectances of Kodak Q<sub>60</sub>, Color checker SG and Munsell data sets from the responses of a digital camera which are contaminated with different levels of random and quantization noises. The root mean square (RMS) error between the reconstructed and actual reflectances as well as the CIELAB color difference values under illuminant A for CIE1964 standard observer are computed to evaluate the performances of the employed techniques. Two different modes of camera responses i.e. three and six channels types are considered. In addition, the responses of a three-channel camera are computed under two sets of illumination conditions to prepare two collections of RGB data. To analyze the performances of the methods, they are also evaluated in the cross media condition, i.e. using different training and testing packages. According to the results, the wPI method, which is the simplest method among the other spectral reconstruction techniques, shows the greatest robustness at different levels of quantization and random noises.

**Key words:** Spectral reconstruction, pseudo inverse (PI), principal component analysis (PCA), canonical correlation regression (CCR), weighted methods, robustness.

## I. INTRODUCTION

DIGITAL imaging devices such as digital cameras and scanners have undergone significant progress in their technology during the recent decades. Today, these modern image capturing devices are widely used as low cost color measuring device in various applications. Nevertheless, the colorimetric output values of these devices are involved with noise in different types and levels. A digital image is subjected to several types of noise sources, that could be fixed, temporal as well as temperature dependent. Noise components can be classified in different ways. One of the most significant noise components is the random noise. Random components include photon shot, reset and thermal noise. The other important noise is quantization

noise caused by analogue to digital (A/D) conversion [1].

While the multispectral image capturing devices use between 3 and 10 optical sensors, the hyper spectral cameras contain as many as 200 (or more) contiguous spectral bands and measure the spectrum for each pixel. The measurement bands of such devices usually extend to invisible spectra of the electromagnetic spectrum, i.e. near infrared and and/or ultraviolet regions [2-6]. Since such devices are still expensive and mostly unavailable, the estimation of spectral data from the camera outputs with limited numbers of channels, e.g. RGB outputs of conventional 3 CCD cameras, has been very attractive subject within the recent years. Through such estimation method, a 3 to 6 device-dependent low dimensional data (such as RGB colorimetric data in a three-channel camera) is converted to device-independent spectral data (like 31 spectral information) by implementation of suitable mathematical and statistical methods. Several methods e.g. pseudo inverse (PI) [7,8], principal component analysis (PCA) [9,10], canonical correlation regression (CCR) [11], independent component analysis (ICA) [12,13]. Fourier bases [14], wavelet bases [15], Gaussian primaries [14], Wiener approach [16-18], matrix R [5], neural network [4,19] and interpolation techniques [4,20] have been introduced to optimize such one to many problems. Besides, the weighted versions of these techniques [21-23] were introduced as improvement to the classical techniques [24,25]. It is a fact that the methods were mostly employed on the XYZ colorimetric data while the RGB information which could be treated with different types of noises has not been considered seriously. Obviously, the awareness of the efficiency of the recovery methods in presence of random and quantization noises could play a significant role in selection of the suitable technique.

In this study, the performances of three prevalent methods in the estimation of spectral data from colorimetric information, i.e. PI, PCA and CCR and their weighted version are compared. The methods are implemented for the recovery of spectral reflectances of different color sets from their corresponding RGB values while they are contaminated with different levels of random and quantization noises. Besides, different training and testing sets are applied to simulate more realistic conditions. Root mean square (RMS) error of the difference between the reconstructed and actual reflectance as well as the CIELAB color difference values under illuminant A for CIE1964 standard observer are reported for the evaluation of the performances of the employed

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methods.

II. EXPERIMENTS

The reflectance spectra of Kodak Q\_60, Color checker SG (CCSG) and the 1269 chips in the Munsell Book of Color Munsell Matt Finish Collection were used to calculate the RGB responses of digital camera. The Gretag Macbeth Eye-One Pro was employed for reflectance measurements of the Kodak Q\_60 color samples. The spectra of Munsell color chips that were measured with a Perkin Elmer Lambda 18 spectrophotometer borrowed from Reference [26]. Besides, the reflectance spectra of 140 samples of Color checker SG were measured by using a Gretag Macbeth Color Eye 7000A spectrophotometer with d/8 geometry. Prior to any calculation, the reflectance data were fixed between 400 to 700 nm at 10 nm intervals.

III. COMPUTATION OF CAMERA RESPONSES

The specimens of each datasets were divided into two groups, i.e. odd and even samples. The odd samples were chosen as training data and used for extracting the eigenvectors, while the even samples were employed for testing purposes. The spectral sensitivities of three and six channels of a typical camera reported by Connah *et al.* [27], were used to calculate the RGB values of the samples from their corresponding reflectance under illuminant D65. Figure 1 shows the employed spectral sensitivities of desired cameras.

Eq. (1) was used to compute the camera responses for each sample.

$$O_i = \sum_{\lambda=400}^{700} R_{\lambda} S_{i,\lambda} E_{\lambda}, \tag{1}$$

where  $O_i$  is the camera responses of each channel  $i$ .  $R_{\lambda}$ ,  $S_{i,\lambda}$  and  $E_{\lambda}$  respectively indicate the spectral reflectance of the sample, the spectral sensitivity of the  $i$ th camera channel, and the spectral power distribution of the light source. To encounter the random and quantization noise effects, the computed camera outputs were subjected to the desired level of proposed noise. In this study, three different levels of quantization noise were implemented in the range of  $\pm \frac{1}{2}$  LSB (least square bit) i.e. 4, 6, and 8 bit quantization levels. Moreover, random noise was added as a small random value which distributed normally with the zero mean and a variable standard deviation (SD) at four levels of 0.00625, 0.0125, 0.025, and 0.05 SD. According to Connah *et al.* [28], reconstruction error increases with increasing the SD of random noise.

Cameras with two different sets of detectors, i.e. the 3 and 6 channels, were examined. In order to calculate the camera responses for each dataset, spectral sensitivities of 3 and 6 channels were respectively implemented. Due to the fact that calculation of spectral sensitivities of six channels needs multispectral cameras which aren't always accessible, we made use of two different illuminants, i.e. D65 and A, to provide a 6 channels outputs from a 3 channels camera. In fact, two sets of RGB values (camera

responses) were calculated under two different illuminants and the collected responses were combined to produce 6 channels outputs. Finally, the cross training procedure was also tried. In other words, Munsell dataset was used as the training package and the other datasets were considered as testing samples which were contaminated with different types and levels of noise.

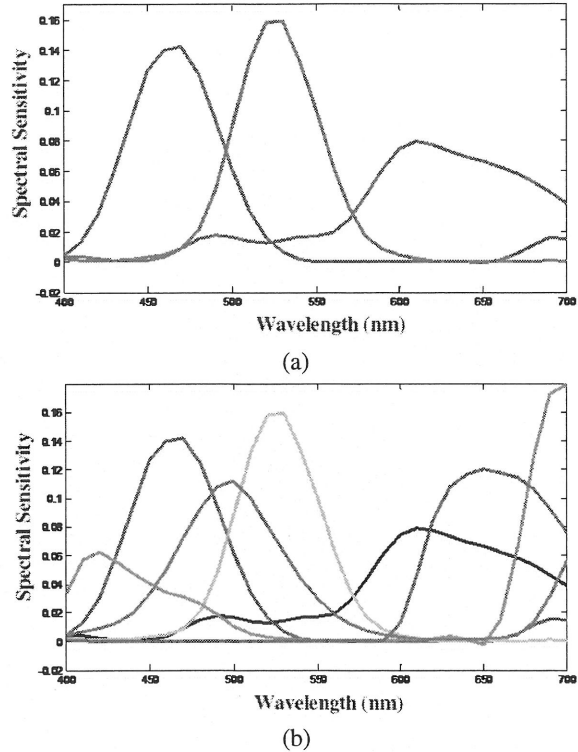


Fig. 1. The spectral sensitivities of three (a) and six (b) channels cameras [27].

IV. EMPLOYED REFLECTANCE RECOVERY METHODS

A. PI and wPI

In this study, the pseudo inverse technique was chosen to evaluate and compare its noise robustness with other common methods i.e. PCA and CCR techniques. Eq. (1) can be rewritten in the matrix form as Eq. (2):

$$O = A^T R, \tag{2}$$

where  $O$  denotes camera responses of sample,  $A$  is the projected matrix obtained from standard illuminant and spectral sensitivity of camera channels, the superscript  $T$  stands for matrix transpose and  $R$  is the reflectance spectrum of the surface [7]. Therefore, for a given set of camera responses  $O$ , the unknown reflectance  $R$  can be simply computed by inversion of matrix  $A^T$ . Since  $A$  is not a square matrix, the matrix inverse would not be possible and the pseudo inverse matrix should be employed. In this method, a transformation matrix,  $M$ , maps the colorimetric values (RGB in the case of three channels camera) to corresponding spectral reflectance which is provided by Eq. (3):

$$R = MO. \tag{3}$$

A set of calibration collections with given colorimetric

and spectral properties were used to optimize matrix  $M$ . As expected,  $O$  is not a square matrix and the generalized inverse is required:

$$M = R \text{pinv}(O), \quad (4)$$

where,  $\text{pinv}(O)$  is the Moore-Penrose pseudo inverse of matrix  $O$  and derived by minimizing the least-square error of the difference between the original reflectance and the estimated one [29]:

$$\text{pinv}(O) = O^T (OO^T)^{-1}. \quad (5)$$

While the standard pseudo inverse method considers equal weights for all spectral data in the main dataset, the method proposed by Babaei *et al.* [23] selectively controls the influence of the reflectance data in the estimated spectrum in the weighted PI (abbreviated by wPI) method. In fact, the wPI method weighs the attending samples in the database according to their color difference values from the proposed sample. So, the wPI method was employed in this study and compared to the other recovery techniques. For a given dataset  $R_{m \times n}$  the weights were introduced in a diagonal matrix  $W$  as:

$$W = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & w_n \end{bmatrix}, \quad (6)$$

where,  $w_i$  refers to RGB color difference values between the samples in the dataset and the sample whose reconstruction was aimed.

$$w_i = \frac{1}{\sqrt{(\Delta R_i)^2 + (\Delta G_i)^2 + (\Delta B_i)^2}}. \quad (7)$$

So, by adding  $W$  in Eq. (3)  $M$  could be achieved by Eq. (8).

$$M = RW \text{pinv}(OW) \quad (8)$$

### B. PCA and wPCA

Many multi spectral imaging techniques utilize the inherent smoothness of reflectance spectra by implementation of low dimensional linear models. Thus, the reflectance spectra,  $R_\lambda$ , sampled at  $k$  equal intervals of wavelength  $\lambda$  may be approximated by a weighted sum of  $m$  basic functions  $B_{i,\lambda}$  with  $i \in 1 \dots m$ , where  $m < k$ , so that in matrix notation:

$$R_\lambda = B_{i,\lambda} V. \quad (9)$$

Therefore, at 10 nm intervals between 400 and 700 nm,  $R_\lambda$  is a  $31 \times n$  matrix of reflectance values of  $n$  samples,  $V$  is a  $m \times n$  matrix of weights, and  $B_{i,\lambda}$  is a  $31 \times m$  matrix of basis functions where column  $i$  contains the  $i$ th basis function  $B_{i,\lambda}$ . Column  $j$  of matrix  $V$  contains  $m$  scalars that provide an efficient representation of the  $j$ th reflectance spectrum. The method proposed by Maloney and Wandell [30] assumes a linear camera model represented by

$$C = M^T R, \quad (10)$$

where,  $C$  is a matrix of camera responses and matrix  $M$  contains wavelength-by-wavelength product of the total spectral sensitivity of the camera and the spectral power distribution of employed light source. If the linear model representation of reflectance is substituted into Eq. (10), then

$$C = \Lambda V, \quad (11)$$

where, matrix  $\Lambda$  represents the product of  $M^T B$ . The surface reflectance factor may be recovered by Eq. (12):

$$V = \Lambda^+ C, \quad (12)$$

where  $\Lambda^+$  denotes the pseudo inverse of the matrix  $\Lambda$  that is known if the spectral sensitivities of the camera's channels, the spectral power distribution of the applied illuminant, and the spectral properties of the basic functions are all known. Once the weighted matrix  $V$  is computed, it is then easy to compute the reflectance spectra from Eq. (9).

According to the study by Agahian *et al.* [21], to extract the principal axes that maximize the weighted variance of the projected data in the eigenvector subspace, the data matrix  $R$  was multiplied with the weighting diagonal matrix  $W$  (Eq. 6) and then, the eigenvector analysis is performed on the weighted covariance matrix.

$$\hat{R} = WR^T. \quad (13)$$

### C. CCR and wCCR

Canonical correlation analysis was developed by Hotelling [31] as a method of measuring the linear relationship between two multidimensional variables. This technique seeks to identify and quantify two bases, one for each variable, that are optimal with respect to correlation [32]. In fact, it focuses on the correlation between linear combinations of two sets of variables instead of one in PCA.

In the spectral reflectance recovery, two sets of variables  $X$  and  $Y$  which respectively represent RGB responses of digital cameras and reflectance data of a set of colored samples could be considered [11].  $X$  and  $Y$  are respectively  $n \times p$  and  $n \times q$  matrices where  $n$  refers to the number of specimens and  $p$  and  $q$  show the number of color coordinates and wavelengths, respectively. The covariance matrix of both datasets could be simply determined by:

$$\begin{aligned} \text{Cov}(X) &= \sum_{i1} , \\ \text{Cov}(Y) &= \sum_{i2} , \\ \text{Cov}(X, Y) &= \sum_{i12} . \end{aligned} \quad (14)$$

As mentioned earlier, the CCA finds two sets of basis vectors, one for  $X$  and the other for  $Y$ , such that the correlations between the projections of the variables onto these basis vectors are mutually maximized. Clearly, in the case of spectral and colorimetric data, the number of wavelengths is greater than the number of colorimetric data ( $p \leq q$ ), so the matrices share the same  $p$  largest eigenvalues,  $\rho_1^2 \geq \rho_2^2 \geq \dots \geq \rho_p^2$ . Canonical correlation

analysis seeks matrices A and B such that the variables U and V maximize the correlation  $\rho = \text{corr}(U, V) = \frac{\text{Cov}(U, V)}{\sqrt{\text{Var}(U)\text{Var}(V)}}$ . So, for coefficient

matrices A and B, two sets of canonical variables could be created as:

$$U = X \times A = X \sum_{11}^{-1/2} E, \quad (15)$$

$$V = Y \times B = Y \sum_{22}^{-1/2} F, \quad (16)$$

while, E and F are the eigenvectors of the matrices  $\sum_{11}^{-1/2} \sum_{12} \sum_{22}^{-1} \sum_{21} \sum_{11}^{-1/2}$  and  $\sum_{22}^{-1/2} \sum_{21} \sum_{11}^{-1} \sum_{12} \sum_{22}^{-1/2}$ , respectively. These two eigenvectors are the normalized canonical correlation basis vectors.

In the reconstruction of reflectance data from the RGB values, matrix X (colorimetric data) is known and the coefficient matrices A and B as well as the canonical correlations between X and Y, i.e.  $\rho$  could be calculated. Hence, the reflectance data of sample ( $Y_{\text{samp}}$ ) which its RGB values ( $X_{\text{samp}}$ ) is known could be easily calculated from estimated canonical variables:

$$\hat{V} = U_{\text{samp}} \times \beta_{\text{cc}} = X_{\text{samp}} \times A \times \beta_{\text{cc}}, \quad (17)$$

where,  $\beta_{\text{cc}}$  is

$$\beta_{\text{cc}} = \text{diag}(\rho_1, \rho_2, \dots, \rho_p), \quad (18)$$

Finally,

$$\hat{Y}_{\text{samp}} = \hat{V} \times B^+. \quad (19)$$

The “+” sign indicates the pseudo inverse of the proposed matrix. The mathematics of classical canonical correlation analysis has been fully described by Johnson and Wichern [33] and its application in spectral reconstruction was discussed by Zhao *et al.* [11] and Eslahi *et al.* [22].

In the standard CCR method all spectral and colorimetric data have equal influence on the reconstruction of spectral reflectances. So, in order to selectively control the influence of the data in the recovery process, both spectral and RGB values of samples attending in the database have been weighted by the inverse of distances of samples in the dataset and the proposed sample in RGB space, i.e. Eq. (7), prior to extraction of canonical terms. In this case, the inputs data, specifically X and Y, change to new variables i.e.  $X_w$  and  $Y_w$  by:

$$X_w = W \times X, \quad (20)$$

$$Y_w = W \times Y. \quad (21)$$

Then, the classical canonical correlation regression could be employed on two sets of weighted variables shown by  $X_w$  and  $Y_w$ .

## V. RESULTS AND DISCUSSIONS

In this study, the performances of PI, PCA and CCR methods and the corresponding weighted version in the estimation of spectral reflectances of samples from their

colorimetric RGB data were examined and compared with each other. Two different levels of colorimetric data were prepared. In fact, the colorimetric data were gathered using 3 and 6-channel cameras. In case of 6-channel camera, two sets of RGB values were used for calculating the weighting matrix in Eq. (7). Besides, in 2-illuminant mode, a three-channel camera was used while the RGB data were collected under two sets of light sources. To investigate the effect of sample type in the training and testing stages, the cross testing-training mode was also examined and the results were compared when identical color set was used in the training and estimation sequences.

### A. 3-channel mode

In this case, the spectral sensitivities of 3 channels of the camera were employed to compute the camera responses for each dataset. The results of estimation of the spectral reflectances of the Munsell, Kodak and CCSG datasets under different levels of quantization and random noises by aforementioned techniques are summarized in Tables I and II. The spectral and colorimetric accuracies of methods were quantified by the calculation of root mean square (RMS) error of the difference between the actual and the estimated spectra as well as the mean of color difference values under illuminant A for CIE1964 standard observer.

As Table I shows, in the absence of any noise, the PI method achieved the best result for all datasets, but the modified version of PCA, i.e. wPCA, led to better result for Munsell package. On the other hand, the weighted versions far surpassed the classic versions, especially in the Kodak dataset whose RMS error decreased from 4.17 to 0.71 by wPCA technique. Also, the weighted versions led to better reconstruction even in the high levels of quantization and random noises in comparison with the classical methods.

However, the PCA and wPCA techniques presented some sort of weakness in comparison to the other implemented methods for the all datasets, especially at high noise levels, such as highest level of random noise (SD=0.05). On the other hand, the wPI and wCCR methods achieved better results even at different types and levels of noise. It means that, these methods are more robust against noise for different datasets with different spectral and colorimetric characteristics. Among the applied datasets, Kodak chart fulfilled better results using the weighted versions in comparison to the Munsell and CCSG databases.

Table III provides the cumulative variance of the Munsell, CCSG and Kodak packages. According to this table, the Kodak chart represents higher cumulative variance which indicates more similarity between the samples. This finding agrees with the fact that the Kodak chart is prepared from a three CMY primaries while the other packages comprise several primaries. Hence, the reconstruction of the spectral reflectances of the samples of this group from the limited number of information, i.e. the RGB values could be more appropriate.



TABLE I  
THE MEAN OF RMS (%) ERROR OF SPECTRAL REFLECTANCE RECONSTRUCTION OF MUNSELL, KODAK AND CCSG DATASETS UNDER DIFFERENT LEVELS OF QUANTIZATION AND RANDOM NOISES FOR 3 AND 6-CHANNEL CAMERAS AS WELL AS 2-ILLUMINANT MODE

Noise type	Noise level	Mode (Channels)	Munsell						Kodak						CCSG						
			PCA	PI	CCR	wPCA	wPI	wCCR	PCA	PI	CCR	wPCA	wPI	wCCR	PCA	PI	CCR	wPCA	wPI	wCCR	
No noise	0	3	2.24	2.07	2.21	1.12	1.24	1.25	4.17	2.76	4.21	0.71	0.59	0.86	2.77	2.58	2.77	1.82	1.72	1.86	
		6	1.22	0.93	1.15	1.29	0.47	0.48	1.12	0.52	1.13	0.14	0.11	0.13	5.36	1.15	1.3	0.69	0.62	0.62	
		2*	-	0.96	1.21	-	0.42	0.43	-	0.56	0.84	-	0.12	0.13	-	1.22	1.38	-	0.63	0.64	
Quantization noise	8 bit	3	2.24	2.08	2.21	1.14	1.26	1.16	4.09	2.77	4.21	0.76	0.63	0.64	3.14	2.59	2.78	2.08	1.75	1.86	
		6	1.38	1.02	1.23	1.67	0.61	0.62	1.2	0.66	1.24	0.43	0.39	0.4	5.72	1.23	1.37	1.3	0.74	0.73	
		2*	-	1.29	1.48	-	0.81	0.81	-	0.98	1.40	-	0.62	0.63	-	1.46	1.63	-	0.98	1.03	
	6 bit	3	2.35	2.18	2.3	1.91	1.47	1.42	4.15	2.83	4.25	1.02	0.89	0.95	3.28	2.73	2.95	2.37	1.88	1.98	
		6	2.19	1.55	1.68	3.56	1.09	1.09	2.15	1.24	1.9	1.81	0.87	0.85	10.11	1.74	1.83	4.38	1.26	1.25	
		2*	-	1.89	2.00	-	1.30	1.27	-	1.67	2.96	-	1.08	1.01	-	2.09	2.21	-	1.50	1.53	
	4 bit	3	3.36	3.08	3.31	3.33	2.7	2.86	4.88	3.61	4.88	3.17	2.22	2.53	4.68	3.56	3.76	4.39	2.87	3.07	
		6	7.18	2.62	2.78	14.35	2.1	2.19	6.9	2.52	3.85	6.39	1.82	1.97	18.31	3.05	3.17	14.04	2.56	2.44	
		2*	-	2.75	2.97	-	2.31	2.38	-	2.84	4.37	-	1.91	2.01	-	3.27	3.38	-	2.63	2.60	
	Random noise	0.00625	3	2.46	2.27	2.38	1.58	1.59	1.52	4.25	2.88	4.3	1.18	1.03	1.17	3.02	2.78	3.03	2.2	2.03	2.19
			6	5.03	1.74	1.86	3.86	1.28	1.23	2.79	1.47	2.31	2.52	1.02	1.03	12.75	1.86	2.07	4.23	1.28	1.48
			2*	-	2.00	2.14	-	1.38	1.35	-	1.81	3.25	-	1.21	1.17	-	2.45	2.44	-	1.79	1.82
0.0125		3	2.97	2.66	2.78	2.3	2.1	2.08	4.54	3.21	4.51	1.89	1.72	1.79	3.48	3.25	3.41	2.82	2.38	2.51	
		6	7.61	2.27	2.34	5.96	1.77	1.68	4.71	2.08	3.42	4.56	1.60	1.55	22.77	2.80	2.63	7.90	2.10	2.02	
		2*	-	2.36	2.49	-	1.81	1.77	-	2.50	3.88	-	1.53	1.61	-	2.85	2.93	-	2.36	2.37	
0.025		3	4.22	3.97	3.75	3.86	3.71	3.19	5.45	4.14	5.1	3.51	3.06	2.99	4.8	4.31	4.39	4.42	3.67	3.56	
		6	13.11	3.03	3.16	8.51	2.59	2.64	8.71	3.05	4.44	8.64	2.52	2.43	44.48	3.51	3.52	16.63	3.10	3.01	
		2*	-	3.00	3.13	-	2.60	2.62	-	3.34	4.59	-	2.32	2.41	-	3.33	3.72	-	2.93	3.11	
0.05		3	7.04	5.67	5.68	6.94	5.23	5.24	7.45	5.8	7.19	6.53	4.83	5.45	7.5	6.18	6.22	7.28	5.46	5.46	
		6	28.89	4.51	4.59	28.03	4.12	4.16	16.34	4.72	5.56	16.30	3.98	4.17	75.79	4.84	5.38	40.37	4.61	5.01	
		2*	-	4.45	4.53	-	4.14	4.04	-	4.75	5.62	-	3.56	4.27	-	5.06	5.03	-	4.74	4.80	

\* Number of illuminant mode

The color differences between the reconstructed and actual spectra of different datasets by different methods are illustrated in Table II. Results of this table are consistent with the results of Table I. As Table II shows, the color difference values are smaller for the PI and CCR techniques in comparison to the PCA method. In fact, the PCA is fundamentally different from PI and CCR methods. Indeed, opposed to the PI and CCR techniques, the most important directions of the spectral reflectance are extracted and ordered by the principal components analysis method and the colorimetric data, i.e. the RGB responses of camera, do not affect the extracted eigenvectors. In other words, the PCA only employs one set of variables (spectral data) while the CCR and PI methods take the advantage of the mutual relations between the colorimetric and spectral data and consequently are more robust in the presence of different types of noise. It is no doubt that the colorimetric data have great impact on the recovery process while they are ignored in PCA technique. So, logically the PCA could lead to suitable results if the noise level would be low enough.

Besides, according to the obtained results, the PI which is the simplest method among the other spectral reconstruction methods is more appropriate than the CCR technique which needs more complicated computation in

the presence of noise.

#### B. 6-Channel mode

The responses of a 6 channel camera were computed from the spectral sensitivities of channels to increase the dimensional properties of the employed data. As Table I illustrates, the accuracy of results are significantly increased by using 6 channels responses for all datasets in different levels of noise for CCR and PI methods and their weighted versions. For example, in the absence of noise, the RMS errors of the CCR method drop from 2.21, 4.21 and 2.77 (for 3-channel camera) to 1.15, 1.13 and 1.30 (for 6-channel camera) for the Munsell, Kodak and CCSG datasets, respectively. Also, the weighted version leads to more precise results than the classic version in absence and presence of noise. It should be noted that the classic PCA technique surpasses its weighted version, wPCA, for the Munsell data set if data has been contaminated with different levels of quantization noise. Furthermore, in the presence of random noise, the PCA technique significantly shows opposite behavior.

TABLE II

THE COLORIMETRIC ACCURACY IN TERM OF MEAN OF  $\Delta E_A$  OF SPECTRAL ESTIMATION OF MUNSELL, KODAK AND CCSG DATASETS UNDER DIFFERENT LEVELS OF QUANTIZATION AND RANDOM NOISES FOR 3 AND 6-CHANNEL CAMERAS AS WELL AS 2-ILLUMINANT MODE

Noise type	Noise level	Mode (Channels)	Munsell						Kodak						CCSG					
			PCA	PI	CCR	wPCA	wPI	wCCR	PCA	PI	CCR	wPCA	wPI	wCCR	PCA	PI	CCR	wPCA	wPI	wCCR
No noise	0	3	1.97	1.87	1.92	1.07	1.18	1.21	4.12	3.23	4.14	0.89	0.79	1.28	2.37	2.27	2.35	1.84	1.64	1.89
		6	0.85	0.72	0.71	1.17	0.44	0.39	1.47	0.65	1.38	0.14	0.15	0.13	8.59	0.86	0.88	0.85	0.51	0.50
		2*	-	0.35	0.37	-	0.19	0.20	-	2.24	0.65	-	1.71	0.09	-	0.70	0.48	-	0.51	0.28
Quantization noise	8 bit	3	2.11	1.96	2.03	1.26	1.33	1.26	4.10	3.27	4.15	1.37	1.14	1.16	3.11	2.40	2.57	2.27	1.67	2.05
		6	1.24	1.00	1.02	1.68	0.80	0.79	1.88	1.20	1.80	1.15	0.95	1.10	8.70	1.28	1.23	1.53	0.90	0.86
		2*	-	1.23	1.21	-	0.94	0.92	-	2.99	2.01	-	2.96	1.17	-	1.17	1.15	-	0.93	1.10
	6 bit	3	3.04	2.93	2.95	2.23	2.49	2.58	4.54	4.26	5.00	3.22	2.75	2.96	4.32	3.74	3.52	3.31	2.90	2.92
		6	3.40	2.46	2.48	4.96	2.10	2.15	5.44	3.59	4.24	5.28	2.58	2.82	15.45	2.79	2.50	4.35	2.50	2.23
		2*	-	2.44	2.42	-	2.02	2.05	-	4.84	4.51	-	4.64	2.59	-	2.62	2.38	-	2.17	2.15
	4 bit	3	8.53	7.20	7.61	8.69	6.94	7.38	13.52	12.40	13.23	13.50	8.39	9.08	11.74	5.78	6.31	10.70	5.20	6.50
		6	12.84	6.28	6.46	28.01	5.49	6.00	23.11	11.26	11.44	22.32	7.25	7.81	39.42	5.68	5.85	39.43	5.55	6.07
		2*	-	6.11	6.67	-	5.85	6.13	-	9.30	9.72	-	9.07	7.58	-	4.90	5.51	-	5.10	5.99
Random noise	0.00625	3	3.70	3.35	3.50	3.26	3.01	3.01	5.42	4.79	5.65	4.50	4.01	4.47	4.60	4.04	4.16	4.31	3.26	3.12
		6	4.59	3.09	3.14	6.08	2.63	0.59	6.58	4.41	5.13	6.35	4.06	4.31	20.73	3.61	3.70	6.06	3.10	3.22
		2*	-	2.86	2.76	-	2.38	2.28	-	5.24	4.82	-	5.00	3.30	-	3.33	3.40	-	2.75	3.58
	0.0125	3	6.30	5.68	5.86	6.06	5.16	5.23	9.04	7.97	8.43	8.87	7.03	6.98	7.29	6.71	7.04	6.87	5.16	6.17
		6	8.94	5.06	4.96	12.03	4.54	4.32	14.24	7.52	8.11	13.70	6.92	7.50	34.69	5.82	5.68	13.67	5.39	4.95
		2*	-	4.30	4.49	-	3.98	4.01	-	6.59	6.84	-	6.29	6.03	-	4.99	5.27	-	4.27	5.35
	0.025	3	12.23	10.80	10.36	12.33	8.96	8.70	17.32	13.51	12.39	18.06	11.11	10.98	12.93	13.75	12.48	12.62	10.64	9.89
		6	21.20	8.43	8.73	17.31	7.48	7.75	26.65	11.41	11.69	26.76	10.05	10.87	58.99	9.69	9.88	30.84	9.16	10.36
		2*	-	7.64	7.66	-	7.01	6.92	-	10.56	9.93	-	9.41	9.86	-	8.05	9.28	-	6.94	8.03
0.05	3	23.44	17.55	16.73	24.09	14.03	13.80	29.97	20.34	20.43	29.60	16.07	17.63	22.82	17.83	18.19	23.31	15.40	15.32	
	6	51.63	14.53	14.02	53.18	12.23	12.11	52.36	17.15	17.32	52.25	14.67	15.47	151.49	14.82	17.92	79.87	14.26	15.97	
	2*	-	13.39	12.91	-	11.69	11.36	-	14.77	15.13	-	13.14	14.45	-	14.01	16.00	-	12.27	15.95	

\* Number of illuminant mode

In comparison with the 3-channel mode, the accuracy of the spectral reconstruction decreases for 6-channel digital camera with the increasing of noise levels. In fact, for the 6 colorimetric camera responses, 6 eigenvectors should be employed to estimate the spectral reflectance of desired sample. However, as shown by Hardeberg [3], the selection of the optimum numbers of eigenvectors is a critical issue in this aspect. In addition, when the spectra are estimated from small numbers of channels, the reconstruction error did not consistently fall with the increasing of channels. In fact, when the PCA technique is used for reconstruction of spectral data, the application of the large numbers of eigenvectors do not grant the achievement of better results because the eigenvectors that have small eigenvalues could include the effect of noises and would make the results less accurate. In other words, 6 colorimetric camera responses need 6 eigenvectors to estimate the spectral reflectance. Obviously, the sensitivity of this method intensifies by increasing the noise and leads to higher RMS errors. Hence, the optimum numbers of eigenvectors should be used for the spectral reconstruction to achieve more accurate results in the presence of high level noises. On the other hand, increasing the camera responses is an advantage for the PI and CCR methods,

since these methods correlate the higher dimension variables which could include the noise. As the results show, RMS errors effectively decrease for these techniques by implementing 6 colorimetric data in all types and levels of noise. In fact, the methods are not significantly affected by the noise type and level. Moreover, for the majority of data at different levels and types of noise, the PI and wPI methods surpass the CCR and wCCR techniques, respectively.

As Table II shows, the color difference values are smaller for PI and CCR techniques in comparison to PCA. Obviously, these methods and their weighted versions imply higher noise robustness, as well. In addition, similar to spectral accuracy, the colorimetric performance of PCA method using 6-channel camera demonstrates inferiority in comparison to 3-channel mode particularly in high levels of noise.

C. 2-illuminant mode

Accessing to a 6-channel camera is not always possible and the conventional cameras are mostly 3-channel ones. To increase the numbers of camera responses, it is possible to capture the images under two sets of illuminations. By this way, 2 sets of camera responses would be possible at a

lower instrument cost. Based on Maloney-Wandell method [30], the PCA faces with technical limitation in the implementation of 6 colorimetric responses because the spectral sensitivities of 6 channels are required for the spectral reconstruction by this method, while a 3-channel camera is used and consequently the spectral sensitivities of them would be available. In contrast, the PI and CCR methods do not have any operational limitations for such type of data capturing technique.

TABLE III  
CUMULATIVE VARIANCES OF MUNSELL, CCSG AND KODAK COLOR SETS

No. Eigenvectors	CCSG	Kodak	Munsell
1	0.8236	0.8246	0.7670
2	0.9531	0.9527	0.9257
3	0.9900	0.9952	0.9856
4	0.9953	0.9975	0.9932
5	0.9979	0.9986	0.9968
6	0.9986	0.9995	0.9980
7	0.9992	0.9999	0.9989
8	0.9996	1	0.9993
9	0.9998	1	0.9996
10	0.9999	1	0.9998
11	0.9999	1	0.9999
12	1	1	0.9999
13	1	1	0.9999
14	1	1	1

As Tables I indicates, in absence of noise, the 2-illuminant mode provides similar results to 6-channel mode and in some cases, such as RMS errors of the wPI and wCCR methods, the values of error respectively decrease from 0.47 and 0.48 for the 6-channel mode to 0.42 and 43 for 2-illuminant mode for the Munsell samples. Besides, as Table II shows, the color difference values of the reconstruction of Munsell data set by the wPI and wCCR techniques diminish from 0.44 and 0.39 (6 channels) to 0.19 and 0.20 (2 illuminants), respectively. Moreover, in the presence of different levels and types of noise, the 2-illuminant technique provides higher noise robustness. Obviously, the 2-illuminant mode benefits from practical advantages in comparison to the 6-channel mode due to the availability and the lower cost.

#### D. Cross training-testing of methods

To evaluate the efficiency of the methods in the more realistic condition, different color sets were employed in the training and testing sequences. In fact, the Munsell dataset was allocated as training samples and the Kodak as well as CCSG samples were used as testing sets. Tables IV and V demonstrate the results of the spectral estimation obtained from the PI, PCA, CCR methods and their weighted versions for 3 and 6 camera channels under different levels of quantization and random noise. It should be noted that the results of the 2-illuminant mode are not reported here because of the large RMS errors.

As shown in Table IV, the PI and CCR techniques and

their weighted versions lead to better results by the 6-channel mode in comparison with 3-channel camera in all types and levels of noise. The spectral reconstruction accuracies of the Kodak chart expose too inferior results in comparison with self-training mode, but CCSG represents different results. In fact, the cross training on CCSG improves the reconstruction accuracy in some cases. For example, according to Tables I and IV the wPCA, wPI and wCCR respectively provide the RMS values of 22.58, 4.30 and 4.37 for the highest level of random noise ( $SD=0.05$ , Table IV) while they lead to the values of 40.37, 4.61 and 5.01 in the self-training mode (CCSG dataset, Table I).

To analyze the results, the colorimetric specifications of the Munsell-CCSG and Munsell-Kodak datasets under D65 illuminant and 1964 standard observer are computed in the CIELAB color space and shown in Figures 2 and 3, respectively. As Figure 2 indicates, the Munsell samples cover the majority of the CCSG specimens, whereas it does not cover the Kodak samples in Figure 3, appropriately. Therefore, the Munsell dataset could not be a suitable chart to use as training set in estimation of the Kodak set. Clearly, CCSG has greater robustness at all types and levels of noise in contrast to Kodak in the both cross and self-training modes. Furthermore, wPI and wCCR show better results among the other applied methods.

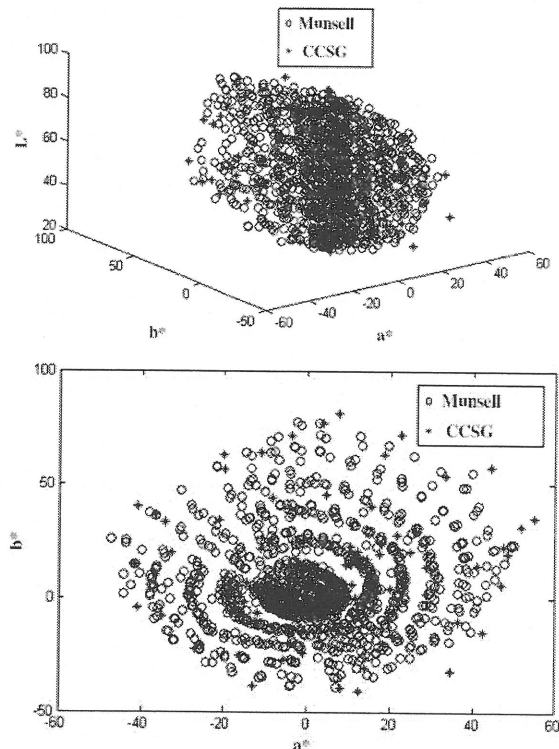


Fig. 2. Three and two dimensional representations of Munsell and CCSG sets under D65 illuminant and 1964 standard observer in the CIELAB color space.

According to Table V, in absence of any noise, the weighted versions surpass the classic ones. Furthermore, the colorimetric accuracies of the 6-channel mode are higher than the 3-channel mode for the PI and CCR

TABLE IV  
THE MEAN OF THE RMS(%) ERROR FOR THE PI, PCA, CCR RECONSTRUCTION METHODS AND THEIR WEIGHTED VERSIONS UNDER DIFFERENT LEVELS OF QUANTIZATION AND RANDOM NOISES FOR CROSS TRAINING MODE

Noise type	Noise level	Mode (channel)	Kodak						CCSG					
			PCA	PI	CCR	wPCA	wPI	wCCR	PCA	PI	CCR	wPCA	wPI	wCCR
No noise	0	3	5.34	5.39	5.37	5.38	5.13	5.23	2.72	2.81	2.71	2.01	1.99	2.03
		6	3.58	3.93	3.43	3.27	3.48	3.52	1.39	1.32	1.24	0.98	0.85	0.89
Quantization noise	8 bit	3	5.34	5.40	5.37	5.38	5.13	5.22	2.73	2.81	2.72	2.03	2.00	1.97
		6	3.61	3.96	3.46	3.29	3.61	3.62	1.48	1.38	1.31	1.18	0.93	0.97
	6 bit	3	5.38	5.42	5.39	5.41	5.19	5.24	2.86	2.97	2.86	2.20	2.20	2.15
		6	4.17	4.33	3.92	3.99	4.20	4.22	2.27	1.93	1.74	2.25	1.44	1.40
Random noise	4 bit	3	6.02	5.89	5.90	6.00	5.77	5.86	4.26	3.65	3.66	3.83	3.35	3.36
		6	8.60	5.26	5.29	6.18	5.18	5.48	7.59	3.11	3.12	4.37	2.68	2.66
	0.00625	3	5.43	5.44	5.43	5.43	5.19	5.28	2.95	3.03	2.89	2.30	2.31	2.20
		6	4.52	4.57	4.20	4.46	4.41	4.41	2.88	2.18	2.01	3.15	1.59	1.55
0.0125	3	5.65	5.63	5.58	5.60	5.40	5.43	3.45	3.48	3.36	2.91	2.86	2.71	
		6	6.30	4.93	4.81	7.24	4.80	4.94	5.11	2.68	2.64	6.11	2.04	2.09
	0.025	3	6.34	6.08	6.09	6.30	5.79	5.88	4.51	4.46	4.18	4.16	3.84	3.54
		6	10.49	5.51	5.45	11.93	5.35	5.49	9.50	3.76	3.50	11.83	3.10	2.93
0.05	3	8.28	7.45	7.39	8.26	7.15	7.09	7.51	5.94	6.25	7.32	5.30	5.73	
	6	18.12	6.64	6.32	20.98	6.37	6.32	18.69	4.80	4.82	22.58	4.30	4.37	

TABLE V  
THE COLORIMETRIC ACCURACY IN TERM OF MEAN OF  $\Delta E_A$  OF SPECTRAL ESTIMATION BY THE PI, PCA, CCR METHODS AND THEIR WEIGHTED VERSIONS UNDER DIFFERENT LEVELS OF QUANTIZATION AND RANDOM NOISES FOR CROSS TRAINING MODE

Noise type	Noise level	Mode (channel)	Kodak						CCSG					
			PCA	PI	CCR	wPCA	wPI	wCCR	PCA	PI	CCR	wPCA	wPI	wCCR
No noise	0	3	4.30	4.10	4.41	4.21	3.68	3.77	2.33	2.27	2.28	1.58	1.55	1.59
		6	3.40	1.03	1.03	2.36	1.03	0.87	1.10	0.87	0.81	0.80	0.59	0.57
Quantization noise	8 bit	3	4.37	4.15	4.48	4.27	3.74	3.84	2.53	2.39	2.46	1.81	1.69	1.70
		6	3.64	1.58	1.61	2.72	1.91	1.92	1.51	1.05	1.11	1.30	0.82	0.87
	6 bit	3	4.90	4.81	5.12	4.85	4.50	4.61	3.87	3.40	3.31	3.21	2.77	2.66
		6	6.21	4.67	4.71	5.74	4.80	4.86	3.70	2.63	2.37	3.72	2.23	2.13
Random noise	4 bit	3	13.48	11.48	12.96	12.91	11.23	12.90	9.90	6.12	6.33	9.58	6.39	7.02
		6	25.60	11.02	12.33	26.85	10.52	12.63	16.54	5.17	5.73	24.24	5.36	5.96
	0.00625	3	5.69	5.44	5.58	5.52	5.19	5.02	4.47	4.22	4.12	4.02	3.55	3.24
		6	7.88	5.33	5.76	8.97	5.22	5.25	5.07	3.48	3.48	5.39	2.73	2.87
0.0125	3	8.04	8.08	8.04	7.79	7.53	7.00	7.27	7.05	7.11	6.88	6.22	5.67	
		6	14.31	8.27	8.20	14.80	7.84	7.21	9.53	5.64	5.17	10.13	4.86	4.25
	0.025	3	14.55	12.30	12.72	14.79	10.89	10.27	11.67	12.72	11.49	11.49	9.50	7.95
		6	29.77	11.67	11.89	29.58	9.91	10.33	21.67	10.68	9.03	21.53	8.82	7.40
0.05	3	26.57	20.25	19.42	26.86	16.51	16.00	25.46	17.35	20.16	26.03	13.13	13.98	
	6	59.32	17.13	16.12	61.37	14.85	13.72	43.38	14.11	14.71	43.51	11.96	12.26	

techniques. In addition, the CCR and PI methods present higher noise robustness in comparison to PCA.

VI. CONCLUSION

Three common techniques, i.e. PI, PCA and CCR and their weighted versions were employed for the estimation of the reflectance spectra of the Kodak Q\_60, Colorchecker SG and Munsell datasets from the colorimetric responses of a virtual camera under different levels of quantization and random noises. The robustness of the implemented methods was evaluated by computation of the root mean square (RMS) error of the difference between the reconstructed and actual reflectance as well as the CIELAB color difference values under illuminant A for

CIE1964 standard observer.

As the results indicate, the PI and CCR methods achieved higher noise robustness in comparison to PCA. In fact, the inherent relation between the two sets of variable in these methods, i.e. colorimetric and spectral data, led to their superiority especially at high level of noise.

In addition, the effect of the numbers of camera responses on the reconstruction error was evaluated by using of the 3 and 6-channel camera modes as well as employing of a 2-illuminant approach. The availability of using a 3-channel camera under 2 illuminants made this method more practical in comparison to 6-channel camera style. Moreover, the type of training dataset showed a significant role in the results of spectral recovery. The



similarity between the training and testing datasets led to the better results. In conclusion, the wPI as the simplest technique among the spectral reconstruction approaches was found the best method for the reflectance recovery in presence of different levels of quantization and random noises.

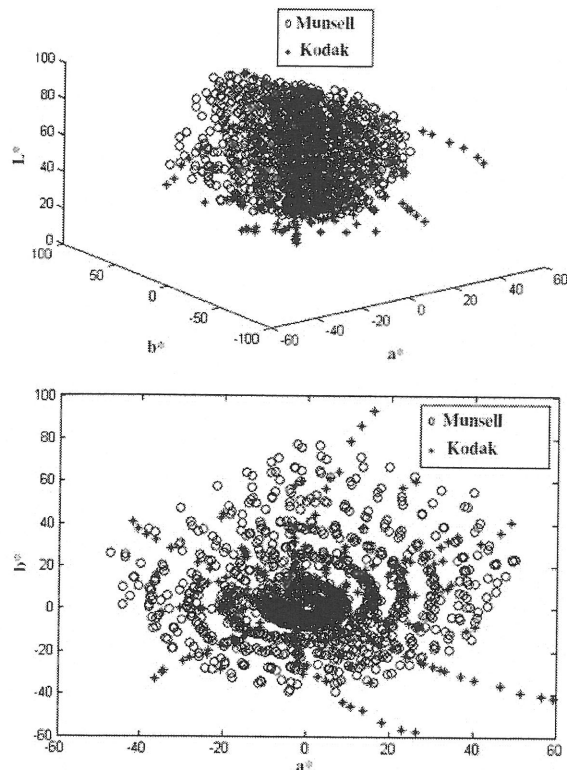


Fig. 3. Three and two dimensional representations of Munsell and Kodak sets under D65 illuminant and 1964 standard observer in the CIELAB color space.

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