Viscoelastic Modeling of the Recovery Behavior of Cut Pile Carpet After Static Loading: A Comparison Between Linear and Nonlinear Models

Sahar Jafari* and Mohammad Ghane

Abstract- **In this work, the recovery property of machine-made carpet was investigated using the viscoelastic models. Viscoelastic modeling of textile materials are usually performed with presenting different combination of spring and dashpot. Two different mechanical models including linear and nonlinear Jeffery's mechanical model were analyzed and compared. Static loading was applied to the samples for a certain time. The load was then removed and recovery behavior was investigated. The experimental data were then adapted to the theoretical data using curve fitting based on least square method. The results showed that there was a reasonably good agreement between the Jeffrey's models and experimental data. The reason is that these two models are not completely elastic showing secondary creep. No significant difference was observed between linear and nonlinear model. The speed of recovery at the removal of the static loading was also analyzed. The results show that the nonlinear Jeffery's model indicates less value of speed of recovery at zero time in comparison to the linear model. The recovery of cut pile carpers shows some permanent creep and is explained by nonlinear and linear Jeffery's model acceptably.**

Keywords**: cut pile carpet, static loading, recovery, viscoelastic models, thickness loss**

I. INTRODUCTION

Standing and walking are different forcesand deformations that applied to Carpet piles during such human daily activities. Axial compression, bending, flattening, and extension and also static and dynamic pressure are some examples of these deformations [1]. Furthermore, the compression behavior and mechanical reaction influence on the piles thickness lossin the carpet. The recovery behavior of the pile carpet after static loading is important for the quality, performance and lifetime of the piled carpet after static loading [2]. The ability of carpet pile to return to its original state after deformation is denoted as Recovery or Resiliency.

For modeling the mechanical behavior of the textile materials, Different combinations of spring and dashpot systems are usually considered. The viscoelastic models are usually consisted of two elements i.e. viscous dashpots which obey Newton's law and springs which obey Hooke's law [3]. However Non-Newtonian dashpots or nonlinear springs are also involved to explain the different aspects of the materials properties. Meng and Wu [4] studied the stress relaxation of membrane structures in the pre-stress state by considering viscoelastic properties of coated fabrics. The validity of the suggested analytical method is examined by comparing the numerical results with experimental data of model tests. Shim *et al.* [5] studied dynamic mechanical properties of fabric armor. The results show that the proposed constitutive equation, based on a linear viscoelastic model is able to describe reasonably accurately the experimental stress-strain response over a range of strain rates.Vangheluwe [6] applied standard nonlinear model to characterize the relaxation and inverse relaxation of yarns after dynamic loading. The results show that the model can calculate the residual force at infinite time in a relaxation test for the tensile curve of spun yams. There seems to be good correlation with the results of tests made using this method.

Manich *et al.* [7] studied the Effect of processing and wearing on viscoelastic modeling of polylactide/wool and polyester/wool woven fabrics subjected to bursting. The viscoelastic behavior of the fabrics when multidirectional extended was simulated and modeled using a modified non-linear Maxwell model. Asayesh *et al.* [8, 9] estimated the creep and fatigue behaviors of plain woven fabrics. They used the Eyring's model and the standard linear model to describe the fabric behavior.

Hashemi *et al.* [10] and Ardakani *et al.* [11] studied the influence of wo bar warp-knitted structure on the fabric

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tensile stress relaxation. They examined different fabric structure and analyzed different viscoelastic models. The experimental results showed that the fabric structure affects the tensile stress relaxation significantly. They also concluded that the three-component model with the parallel-connect nonlinear spring shoed the best agreement with the experimental relaxation of the analyzed fabrics.

Khavari and Ghane [12] studied the compression and recovery behavior of cut pile carpets under constant rate of compression by mechanical models. Maxwell mechanical model, the standard linear and nonlinear models were employed for simulating the compression behavior and recovery of the carpet samples. The results showed that the standard nonlinear models can describe the compression behavior of more significantly in compression to Maxwell and standard linear models. Jafari and Ghane [13] studied the recovery behavior of cut pile carpet after static loading by mechanical models. Jeffery's model and standard linear model was used for simulating the recovery behavior. The standard linear model showed poor regression for the recovery properties of cut pile carpets after static loading. The reason was that the standard linear model was completely elastic. Jafari and Ghane [14] also studied the effect of UV radiation on the recovery properties of pile carpet after static loading through analytical and viscoelastic modeling. They found that the thickness loss and the maximum compression were both higher at longer UV exposure times.

The aim of this work is to investigate the the oretical and experimental recovery behavior of the cut pile carpets. In this study, two well-known mechanical models i.e. the linear and nonlinear Jeffery's model were analyzed. To find the best model semi empirical curve fitting method based on the least square method was used.

II. MECHANICAL MODELING

Different mechanisms are involved when a cut pile carpet is exposed to compressive force. The first one is aviscoelastic deformation. The deformation is recoverable with a retardationtime. The second mechanism involves a plastic or non-recoverable deformation. This is due to dispassion of energy between the pile yarns and plastic deformation of the constituent fibers. It seems that mechanical models consisting of different combination of spring and dashpot can be used to represent the elasto-plastic behavior of the carpet under compressive static load.

The aim of this paper is to analyze and compare different viscoelastic models in order to explain the mechanical behavior of carpet piles after a short-term static loading. For this purpose, two models were presented for simulation

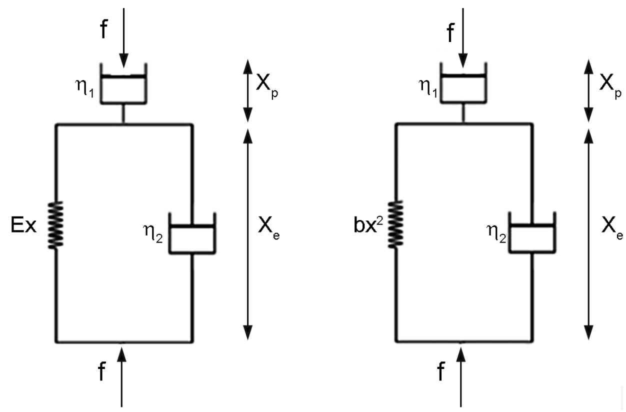


Fig. 1. Viscoelastic models for the recovery behavior of cut pile carpet: (a) linear Jeffery's model and (b) nonlinear Jeffery's model.

of recovery behaviors of carpet pile yarns including; a) linear Jeffery's model [15], b) quadratic nonlinear Jeffery's model. Schematic diagrams of two models are presented in Fig. 1. The compressive force f is applied to the model. Erepresents the spring constant for linear model (N/m) and b is the spring constant for nonlinear model and η is the dashpot constant (N.s/m).

The governing differential equation for the linear viscoelastic model is found to be:

$$\text{a)}\ \dot{f}\left(1+\frac{\eta_2}{\eta_1}\right)+\frac{E}{\eta_1}f-(E\dot{x}+\eta_2\ddot{x})=0 \quad (1)$$

In this modeling a constant load (f_c) is applied to the model. Thus, to find the x-t equation for the nonlinear model, we can separate two part of extension.

$$x = x_e + x_p \quad (2)$$

We can calculate plastic deformation x_p and elastic deformation x_e separately and then add them together. In plastic part we have:

$$f = \eta_1 \frac{dx_p}{dt} \quad (3)$$

In the case of constant load (f_c), we have:

$$x_p = \frac{f_c}{\eta_1} t \quad (4)$$

The governing deferential equation in elastic part is:

$$\dot{f} = 2bx_e\dot{x}_e + \eta_2\bar{x}_e \quad (5)$$

In the case of constant load it becomes:

$$2bx_e\dot{x}_e + \eta_2\bar{x}_e = 0 \quad (6)$$

For the elastic part we have the following boundary conditions:

$$x_e = 0 \text{ at } t = 0 \text{ and } x_e = \sqrt{\frac{f_c}{b}} \text{ at } t = \infty \quad (7)$$

Solving the differential Eq. (6) and applying the boundary condition as in Eq. (7) the deformation of the elastic part under the constant load is found to be:

$$x_e = \sqrt{\frac{f_c}{b}}\ \tan h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t\right) \quad (8)$$

For the nonlinear model the total extension under the constant load can be found by summing Eqs. (4) and (8). The x-t equations for the linear (model a) and quadratic nonlinear model (model b) are shown in Eq. (9):

$$\text{a)}\, x = \frac{f_c}{E}\left(1 + e^{\frac{-E}{\eta_2}t}\right) + \frac{f_c}{\eta_1}t$$

$$\text{b)}\, x = \sqrt{\frac{f_c}{b}} \tan h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t\right) + \frac{f_c}{\eta_1}t \quad (9)$$

The constant load, f_c, is applied to the proposed model. The set is compressed to the extent of x_1 at the time t_1. The value of x_1 is found to be as in Eqs. (10):

$$\text{a)}\, x_1 = \frac{f_c}{E}\left(1 - e^{\frac{-E}{\eta_2}t_1}\right) + \frac{f_c}{\eta_1}t_1$$

$$\text{b)}\, x_1 = \sqrt{\frac{f_c}{b}} \tan h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t_1\right) + \frac{f_c}{\eta_1}t_1 \quad (10)$$

It is clear that the compression of the whole set is equal to the compression of both elastic (x_{e1}) and plastic parts (x_{p1}) i.e. $x_1=x_{e1}+x_{p1}$. Thus, the amount of compression for elastic and plastic parts can be determined separately as in Eqs. (11):

$$\text{a)}\, x_{e1} = \frac{f_c}{E}\left(1 - e^{\frac{-E}{\eta_2}t_1}\right) \text{ and } x_{p1} = \frac{f_c}{\eta_1}t_1$$

$$\text{b)}\, x_{e1} = \sqrt{\frac{f_c}{b}} \tan h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t_1\right) \text{ and } x_{p1} = \frac{f_c}{\eta_1}t_1 \quad (11)$$

After removing the applied constant load from the set, recovery equations could be obtained. By considering the general equation of the models as well as our condition (after load removal; f=0, f=0) the recovery differential equations for both models are achieved as follows:

$$\text{a)}\, E\frac{dx}{dt} + \eta_2\frac{d^2x}{dt^2} = 0$$

$$\text{b)}\ 2bx_r\frac{dx_r}{dt} + \eta_2\frac{d^2x_r}{dt^2} = 0 \quad (12)$$

Solving these differential equations need boundary conditions. The conditions can be provided from the displacement equation where $x=x_1$ at $t=0$, as well as $x = x_{p1}$ at $t = \infty$. For the nonlinear model we consider $x_r=0$ at $t=0$ and $x_r=x_{e1}$ at $t=\infty$. Thus, thickness loss-time equations obtained as in Eqs. (13) or (14):

$$\text{a)}\ x = x_{e1}e^{\frac{-E}{\eta_2}t} + x_{p1}$$

$$\text{b)}\ x_r = x_{e1}\tan h\left(\frac{b.x_{e1}}{\eta_2}.t\right) \text{ and } x = x_1 - x_r \quad (13)$$

Or; replacing the boundary values form Eqs. 10 and 11 gives:

$$\text{a)}x = \frac{f_c}{E}\left(1 - e^{\frac{-E}{\eta_2}t_1}\right)e^{\frac{-E}{\eta_2}t} + \frac{f_c}{\eta_1}t_1$$

$$\text{b)}\, x = \left[\sqrt{\frac{f_c}{b}}\tan h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t_1\right) + \frac{f_c}{\eta_1}t_1\right] - \left[\sqrt{\frac{f_c}{b}}\tan h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t_1\right)\tan h\left(\frac{\sqrt{b.x_{e1}}}{\eta_2}.t\right)\right] \quad (14)$$

III. MATERIALS AND EXPERIMENT

A. Materials

In this work, the 700-reeds machine woven carpet samples with 700×800 densities of knots were produced. The carpet samples consist of the cotton/polyester blended yarns with density of 700 per meter as the warp yarns, the jute yarns with density of 800 per meter as the weft yarns. Acrylic yarn was used as the pile yarn and the pile height was adjusted to be 10 mm.

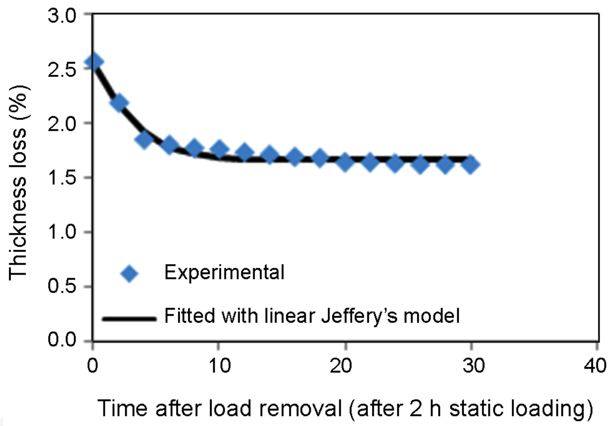
B. Experiments

Digital thickness gauge was used to measure the sample thickness. Initial thickness of the sample was measured and denoted by x_0. A laboratory scaled static loading apparatus was used to simulate the applying of the static loading. The appropriate weight was hanged on the end of the arm of the apparatus so that the constant pressure of 700 kPa was applied to the carpet sample. The load was then removed after the time t_1. Two levels of static loading (t_1) were tested i.e. 2 min and 2 h. The thickness of the carpet is measured at this time (x_f) which is considered as zero time for the recovery mode. The compression of the carpet at the time t_1 (x_1) is denoted by $x_1=x_0-x_f$. The recovery process was then monitored and in every 2 min intervals the thickness of the carpet (x_t) is measured. This continues up to 30 min. The thickness loss of the samples at the time t is then considered as $x=x_0-x_t$.

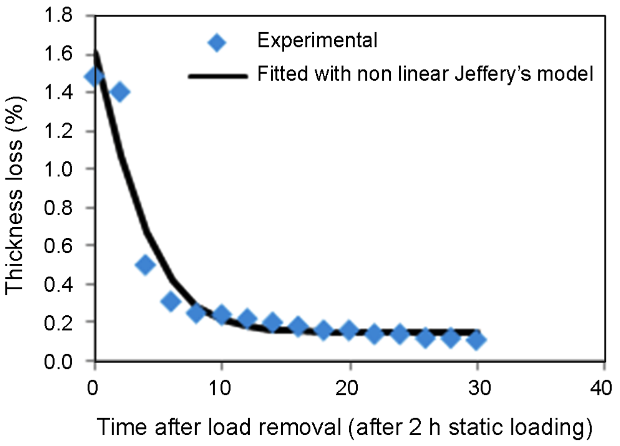
IV. RESULTS AND DISCUSSION

In this paper, the recovery property of the pile carpets after static loading was investigated by presenting two viscoelastic models. Analytical approach was used and the recovery-time equations were derived. The theoretical equations were then adapted to the experimental data using best curve fitting based on the least square method. Determination of the best fit equation allows the extracting the discrete viscoelastic parameters of the models.

The recovery behavior of the pile carpet was investigated in two different static load time i.e. 2 min and 2 h. In each level of static loading, the values of thickness loss (x) were measured in the interval times of every 2 min.



(a)



(b)

Fig. 2. Fitted curves vs. experimental data for recovery after 2 h and 2 min static loading: (a) linear Jeffery's model and (b) nonlinear Jeffery's model.

Fig. 2 shows the experimental data and the fitted curves for recovery behavior of the carpet pile yarns after applying 2 min and 2 h static loading.

Fig. 2 reveals that there is a good correlation between the fitted and the experimental data for two Jeffery's model. The fitted curves show very high R-square of 0.977. This can provide a suitable explanation for the recovery behaviors of the carpet. The model is able to explain the non-recoverable deformation of the carpet piles due to static loading. The model is also describes the effect of the time of the constant static loading (See Table I).

Eqs. (15a) and (15b) representthe equations of the fitted curve with linear Jeffery's model and nonlinear Jeffery's model respectively.

$$\text{model (a)}\left|\begin{array}{l} 2\ \text{hours}: x = 0.91e^{-0.295t} + 1.66 \\ 2\ \text{minutes}: x = 1.49e^{-0.253t} + 0.12 \end{array}\right. \tag{15a}$$

$$\text{model (b)}\left|\begin{array}{l} 2\ \text{hours}: x = 2.56 - 0.90\ \tanh\ (0.225.t) \\ 2\ \text{minutes}: x = 1.60 - 1.46\ \tanh\ (0.191.t) \end{array}\right. \tag{15b}$$

To calculate the model parameters the constant load, f_c, is needed. This was calculated from the pile densities and pressure load applied by the laboratory-scaled static loading apparatus. The constant load, f_c, applied to each pile was calculated as 1.4 N/pile.

The model parameters were determined using fitted Eqs. (15a) and (15b) in accordance to Eq. (14). The results are shown in Table I.

The good agreement between the fitted curve and the experimental data suggests that two Jeffrey's model can predict well the recovery behavior of carpet piles after static loading. Analysis and comparison between two models revealed that there was no significant difference between the linear and quadratic nonlinear models in order to justify the recovery properties of the cut-pile carpet after removing of the static loading. It can be concluded that the Jeffery's linear model shows significant accuracy and can be proposed to explain the recovery properties of the cut-pile carpets.

Another outcome of the results is that both spring constant and dashpot constant show less values in longer static loading. This shows clearly that the time affects the fine structure of the fibres. In other words, the time causes deterioration in the crystal regions as well as the amorphous regions. This suggestion needs more fine structural analysis.

VI. SPEED OF RECOVERY

The speed of recovery at the start of recovery process can

TABLE I
ESTIMATED VISCOELASTIC PARAMETERS

Model	Time	R^2	Spring constant	$\eta_1\left(N.\frac{min}{mm}\right)$	$\eta_2\left(N.\frac{min}{mm}\right)$
a) linear	2 min	0.961	E=0.37 (N/mm)	22.58	1.46
Jeffery's model	2 h	0.977	E=1.54 (N/mm)	101.57	5.22
b) nonlinear	2 min	0.956	b =0.22 (N/mm^2)	20.00	1.68
Jeffery's model	2 h	0.972	b =1.75 (N/mm^2)	100.77	6.95

be a good criterion to evaluate the compression properties of the pile carpets. To find the speed of recovery, the first derivation to recovery-time equation (Eq. 14) can be calculated as shown below:

$$a)\ \dot{x} = -\left(\frac{f_c}{\eta_2}\right)\left(1 - e^{\frac{-E}{\eta_2}t_1}\right)e^{\frac{-E}{\eta_2}t}$$

$$b)\ \dot{x} = -x_{e1}\frac{\sqrt{b.f_c}}{\eta_2}\tan\ h\left(\frac{\sqrt{b.f_c}}{\eta_2}.t_1\right)\sec h^2\left(\frac{b.x_{e1}}{\eta_2}.t\right) \quad (16)$$

Applying the values of the parameters in to Eq. (16) the speed of recovery at zero time for both linear and nonlinear model were calculated as can be seen in Table II. The experimental values of the speed of recovery at zero time were calculated form the slope of tangent to the displacement-time curves (Fig. 2) at zero time (t=0). The results of the theoretical and experimental values of the speed of recovery at zero time are shown in Table II.

As seen in Table II, the speed of recovery at the start of recovery (zero time) is less in longer static loading. The reason may be explained by the deterioration or degradation of the fine structure of the fibers in longer time of static loading. The nonlinear model also shows lower recovery speed in 2 min as well as 2 h static loading. Comparison of the theoretical and experimental values of the speed of recovery at zero time reveals that two values are reasonably close to each other. Furthermore, the results of theoretical values by the nonlinear model are closer to the experimental values. It can be concluded that the nonlinear model is more reliable to predict the speed of recovery in comparison to the linear model.

The recovery properties of the cut pile carpets is affected by different factors including the type and mechanical properties of pile yarn and weight and duration of static load. Finding an appropriate mechanical model and calculating the viscoelastic parameters allows investigating on the effect of different effective factors on the recovery properties of the cut pile carpets. Finally, we may be able to propose the structure and materials to gain optimum recovery properties.

VI. CONCLUSIONS

The aim of this paper was to compare two viscoelastic models to describe the recovery behavior of the pile carpet after static loading. Using analytical procedure recovery-time equations were derived and compared to the experimental data. Best fitted curves based on the least square method were determined. The models were analyzed and a comparison was made between the linear and quadratic nonlinear Jeffry's models. Using the analytical methods the recovery equations were derived and the district viscoelastic parameters were determined and presented. The result of curve fitting procedure showed that both linear and nonlinear models fit the experimental data significantly. However, no significant difference was observed between two models. The nonlinear model provided less speed of recovery at the start of the recovery

TABLE II
SPEED OF RECOVERY AT ZERO TIME FOR LINEAR AND NONLINEAR MODEL FOR RECOVERY

Model	Time	Speed of recovery (mm/min)	Speed of recoveryat t=0 (mm/min)	
			Theoretical	Experimental
a) linear	2 min	$\dot{x} = -0.38e^{-0.253t}$	0.38	0.32
Jeffery's model	2 h	$\dot{x} = -0.27e^{-0.295t}$	0.27	0.22
b) nonlinear	2 min	$\dot{x} = -0.28\sec h^2(0.191t)$	0.28	0.27
Jeffery's model	2 h	$\dot{x} = -0.20\sec h^2(0.225t)$	0.20	0.19

mode in comparison to the linear model. The results of the analysis also revealed that the model parameters i.e. the spring consents as well as the dashpot constants are significantly less in longer static loading time. It may be concluded that the term of static loading affects the internal structure of fiber. Deterioration of the internal structure can lead to less value of model parameters. More fine structure analysis is needed to verify this conclusion.

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