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ORIGINAL PAPER

Role of Fabric Structure on the Second Tensile Elastic Modulus of Net Warp-Knitted Fabrics

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***Abstract*- This work aimed to study the second elastic modulus of net warp-knitted fabrics (NWKF) experimentally and theoretically. Net knitted fabrics were selected to study owing to their wide practical application and also lack of enough attention. Knitted fabrics have two regions of tensile elastic behavior which the initial one refers to the displacement of the elements of fabrics before jamming of the yarns. As soon as jamming takes place completely, the constituent yarns in the structure of fabrics go through tension. In other words, yarns in their spatial configuration resist the applied loads. This resistance is important in the ultimate strength of the fabrics and also in the composite materials reinforced with knitted fabrics. NWKFs were produced using a Raschel knitting machine. Uniaxial tensile tests were conducted in the course and wale directions. The second elastic modulus was measured using a statistical approach. In the theoretical part, a mechanical model based on the configuration of elements of fabrics after jamming status and using Energy method and Castigliano's Theorem was proposed. The proposed model was used to calculate the second elastic modulus. To validate the model, the authors' experimental data along with the collected data from the other researches were compared with the calculated values, and the results showed that the proposed model can predict the modulus reasonably. The results showed that the fabric structure including wale spacing, course spacing, and the number of lapping movements creating the holes affects the second elastic modulus despite the accepted concept, but the mechanical properties of yarns only influence the second elastic modulus of the fabrics.**

***Keywords*: net warp-knitted fabrics, second elastic modulus, uniaxial tests, energy method, castigliano theorem**

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I. INTRODUCTION

Warp-knitted fabrics have been found various applications in different industries and the demand for them has continued to rise. They generally have applications in civil and construction, automotive, and aerospace. Moreover, light-weight composites and medical applications like elastic knitted bands that are widely used in rehabilitation and prophylactic goods are the other usages [1-3]. One of the most advantages of the warp-knitting technique is that net fabrics can be produced simply and without any special equipment. Furthermore, knitting is particularly suitable for the rapid manufacture of components with complex shapes due to the low resistance to deformation of knitted fabrics [4]. In addition to apparel and household textiles, net warp-knitted fabrics (NWKFs) are usually used for preformed composite materials whether using cement or resin matrices. Hence, the study of the physical and mechanical properties of these kinds of fabrics is important. Although net warp-knitted fabrics have many practical applications, there are seldom in-depth studies of their characteristics. Different fabrics have various structures that have been made from different materials. The knowledge of the material is extensive and therefore it is necessary to recognize the geometry of fabrics carefully to investigate their behavior. It should be noted that the fabrics are structures that can resist and transmit applied loads [5]. There are a few research studies in the field of tensile properties of warp-knitted fabrics due to their structural complexity [1,6-12]. However, compressive properties of warp-knitted fabrics, especially spacer ones have been the subject of many pieces of researches [13-15]. In recent years, a few papers have been published to determine the spatial configuration and geometry of knitted fabrics using computer modeling which helps researchers to simulate fabrics and study their behavior before production to prevent waste of time, cost, and effort [16]. Most of these researches are about common knitted fabrics and net fabrics are rarely considered in the literature. When a fabric is subjected to uniaxial forces, two elastic regions can be considered in their load-extension diagrams. It is important to note that there is an unstable transition region between them. The initial elastic region refers to the displacement of the fabric structure in the direction of the applied load. In other words, the initial elastic behavior is the result of the deformation of loops in the direction of the applied load until jamming of yarns takes place. After the jamming happened, the constituent yarns go through tension and the behavior of the fabric as a structure is the sum of the resistance of the yarns. After the second elastic region, yarns were stretched and the breakage of the fabric happened as a result of yarn breakage. Most of the papers address the initial elastic behavior, however, the second elastic behavior is very important for the investigation of composite materials reinforced with knitted fabrics.

As it was revealed, a rare publication is available on theoretical and experimental studies on the tensile properties of net warp-knitted fabrics. Furthermore, the second linear modulus is rarely taken into account. Therefore, this work is intended to investigate the second elastic behavior of net warp-knitted fabrics with two approaches. In the experimental part, NWKF was produced and uniaxial tests were conducted. In the theoretical part, a mechanical model was proposed using Energy method and Castigliano's theorem to calculate the second elastic modulus.

TABLE I
CHARACTERISTICS OF THE USED YARN

Yarn type	Linear density (tex)	Elastic modulus (gf/tex)
PET spun yarns	18.5	242.9

II. EXPERIMENTAL

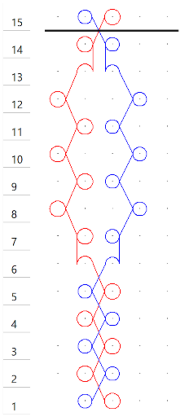
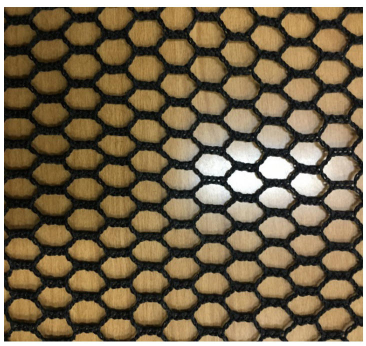
A. Fabric Preparation

A Rachel knitting machine with the gauge eight and two-guide bar was used for producing the net fabrics. Commercial staple PET yarns with negative let-off motion were used for fabric production. The characteristics of yarns are shown in Table I. The lapping movement, produced fabric, and its physical and dimensional properties are shown in Table II.

B. Tensile Tests

To measure the second elastic modulus of the produced fabrics, uniaxial tensile tests were conducted in the course and wale directions. Since there are scarce approved code and methods for determining the specimen size and test speed, samples were examined using the suggested approach [18]. According to the considerations in the used approach, the tests were carried out with a test speed of 50 mm/min, specimen width of 70 mm, and a gauge length of 150 mm. The tensile tests were repeated five times for each specimen.

TABLE II
CHARACTERISTICS OF THE PRODUCED FABRIC

Fabric	Lapping movement[1]	CPC[2]	WPC[2]	Weight[3] (g/m²)	Produced fabric
Net warp-knitted		11.42	3.01	73.08	

[1] The lapping movements were created using the TexMind Warp Knitting 3D software [17].
[2] The values were measured according to ASTM D8007-15e1.
[3] The values were measured according to ASTM D3776-96 (2002).

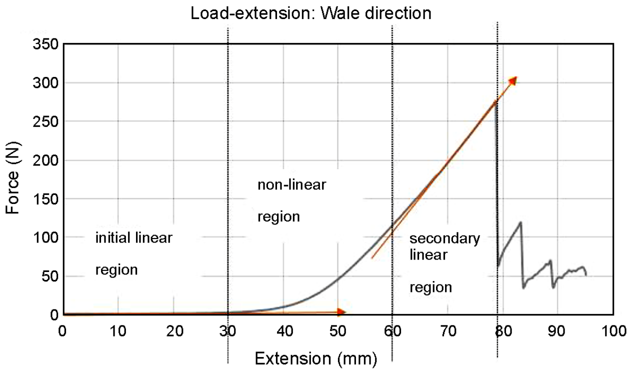


Fig. 1. Typical load-extension diagram of NWKF.

C. Theory

As it was mentioned before, knitted fabrics have two regions of elastic behavior with an unstable transition area between them (Fig. 1). When a fabric is subjected to the action of forces, the geometrical deformation will happen up to the jamming state of the constituent yarns. After jamming takes place, two situations will be arisen based on the structure of fabrics: a) the oblique sides of hexagonal in the determined unit cell of the fabric (Fig. 2a) which become straight after the jamming state, will be under the tension and the constituent yarns of loops creating the sides of hexagonal will deform; b) the yarns creating the loops will resist the applied load and now the elastic modulus of yarns is the dominant factor. The situation "a" happens in the present work, i.e. the sides of hexagonal which were oblique become straight and will be under the forces and this is the situation which refers to the second elastic modulus of the fabrics

When the fabrics are subjected to the action of forces and after the jamming state of the yarns, the shape of the structure of fabrics changes. The hexagonal shape of the holes becomes rectangular during the application of course-wise forces. However, the holes almost become closed and their sides are aligned with the wale-wise loads. Figs. 3 and 4 show the configuration of the unit cell (the hole) and its sides while the second elastic region begins to happen during the application of forces in the course and wale directions, respectively.

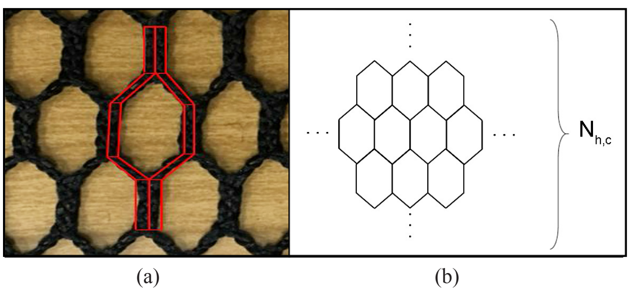


Fig. 2. Structure of the produced fabric: (a) unit cell and (b) the number of holes in the course direction.

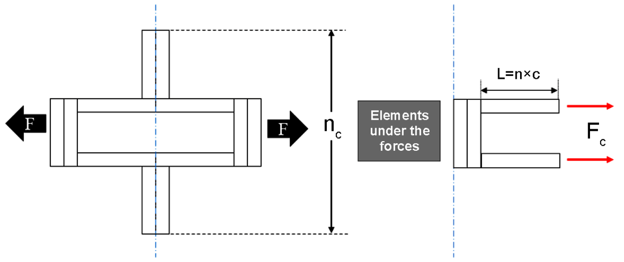


Fig. 3. The unit cell of the fabrics in the course-wise forces after jamming of the yarns: (a) unit cell aligned in the direction of forces and (b) elements used for development of the model.

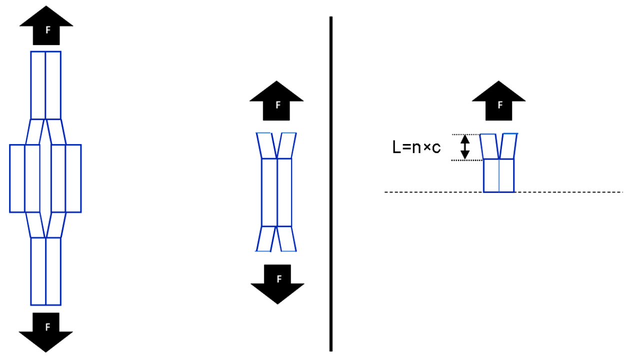


Fig. 4. The unit cell of the fabrics in the wale-wise forces after jamming of yarns: (a) unit cell aligned in the direction of forces and (b) elements used for development of the model.

Based on the straight-line model for two-guide bar knitted fabrics [6], the shape of loops before and after the action of the course-wise applied loads is shown in Fig. 5. When a given fabric which has been under the action of forces in the course direction reaches the jamming condition, then the structure will be in forces. To propose the models, the following assumptions were made:

1. Yarns have circular cross-sections;

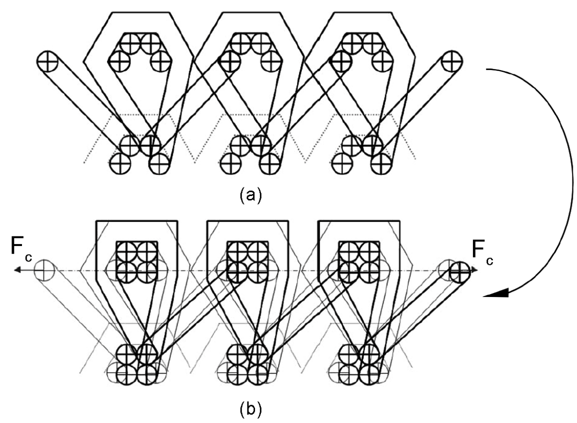


Fig. 5. Shape of the loops: (a) before action of forces and (b) after action of forces [6].

2. Yarns are inextensible and incompressible;
3. Friction between the yarns is neglected.

Assuming the geometry of the fabrics and the determined unit cell, we have:

$$F_{u,c} = \frac{F_{T,c}}{N_{h,c}} \tag{1}$$

$$N_{h,c} = n_c.c \tag{2}$$

In which, $F_{u,c}$ is the force applied to the unit cell in the course direction, $F_{T,c}$ is the force applied to the fabric in the course direction, $N_{h,c}$ is the number of holes counted in the course direction, n_c is the number of course spacing in a hole, and c is the course spacing (1/course per centimeter). It is well known that the fabric extension in course direction (ε_c) is equal to:

$$\varepsilon_c = \frac{\delta W}{W} \tag{3}$$

Where,

$$W = n.c \tag{4}$$

In which W is the wale spacing (1/wale per centimeter), δW is the change in the wale spacing, and n is the number of course spacing. Since the oblique sides of the unit cell became straight as a result of the applied load (secondary linear region), the direction of the sides was changed and the loops creating the sides are horizontal now. Therefore, W is equal to the multiplication of n and c. W can be calculated based on the variation of the direction of sides of hexagonal which are now straight and their lengths are in the size of several course length depending on the lapping movements. Considering the elastic behavior and Hook's law, we have:

$$\delta W = \frac{\sigma_c}{E_c}.n.c \tag{5}$$

The variation of W can be obtained using Castigliano's theorem too. It is equal to:

$$\delta W = \frac{\partial U}{\partial F_{T,c}} = \frac{\partial U}{n_c.c.\partial F_{u,c}} \tag{6}$$

In which, U is the elastic strain energy stored in the fabrics subjected to the axial loads in the course direction.

Equating the Eqs. 5 and 6, we have:

$$\frac{1}{E_c} = \frac{w.d}{n_c.n.c^2.F_{u,c}} \times \frac{\partial U}{\partial F_{u,c}} \tag{7}$$

Where,

$$F_{u,c} = \sigma_c.(w.d) \tag{8}$$

Where, d is the yarn diameter. There is an unknown term, U, in Eq. (7) which can be ascertained using Energy method. When the fabrics are under the axial force in the course direction, tensile stress has the dominant effect and therefore strain energy of elements can be calculated as follows:

$$U = \int_0^L \frac{\sigma_c^2.A.dx}{2E_{yarn}} \tag{9}$$

By integration of U along the length of the straight elements, the strain energy can be concluded:

$$U = \frac{n_c^2.c^2.F_{u,c}^2.L}{2.A.E_{yarn}} \tag{10}$$

Where,

$$\sigma_c = \frac{n_c.c.F_{u,c}}{A} \tag{11}$$

In which A is the cross-section of the specified area of the unit cell under the tensile forces.

Substituting Eq. (10) into Eq. (7) and by differentiation of U with respect to $F_{u,c}$, the modulus of the second elastic region can be calculated as follows:

$$\frac{1}{E_c} = \frac{n_c.w.d.c}{A.E_{yarn}} \tag{12}$$

When the fabric is extended, the shape of loops which forming the structure changes as it was shown in Fig. 5, and then:

$$A = \frac{4.\pi.d^2}{4} \tag{13}$$

Eq. (13) is in accordance with the change in the size of the loops head owing to the application of the forces (Fig. 5). The area which is under the load is four times the area of yarn-cross-section, consequently:

$$\frac{1}{E_c} = \frac{n_c.c.w}{\pi.d.E_{yarn}} \tag{14}$$

A similar procedure can be followed to find the second elastic modulus in the wale direction. There is a marked difference between ascertaining the second elastic modulus in the wale and course directions and it is the constrained elements aligned in the direction of tension. In other words, the area which is under the forces in the wale direction is more than twice the size of the area which was under the forces in the course direction. Using the above-mentioned theory, we have:

$$\frac{1}{E_w} = \frac{w.d}{n_w.n.c.w.F_{u,w}} \times \frac{\partial U}{\partial F_{u,w}} \tag{15}$$

For tensile strain energy in the wale direction, it can be written:

$$U = \int_0^L \frac{\sigma^2.A.dx}{2E_{yarn}} \tag{16}$$

Integrating U along the length of elements, we have:

$$U = \frac{n_w^2.w^2F_{u,w}^2.L}{2.A.E_{yarn}} \tag{17}$$

Where,

$$\sigma_w = \frac{n_w.w.F_{u,w}}{A} \tag{18}$$

Substituting Eq. (18) into Eq. (15) and by differentiation of U with respect to Fu,w, the second elastic tensile modulus in the wale direction can be obtained as follows:

$$\frac{1}{E_w} = \frac{n_w.w^2}{\pi.d.E_{yarn}} \tag{19}$$

D. Other Test Data

The capability of a generally accepted model is that it could deal with a wide range of cases in its studying area. In the present work, the theoretical model was proposed based on the fabric which was produced by the authors. To verify the ability of the model for application in the other kinds of net fabrics, the authors were fortunate to get raw test data of the tensile test of net fabrics from a detailed experimental study [19,20]. The figures and also the physical properties of the fabrics are given in Table III.

III. RESULTS AND DISCUSSION

The second elastic behavior of net warp-knitted fabrics was considered using two approaches namely experimental and theoretical. In the experimental part, uniaxial tensile tests were carried out in the course and wale directions. The second elastic modulus was measured for each test from load-extension diagrams using a statistical method [21]. To determine the tensile moduli values based on the statistical method, the slopes of the load-extension curves at each point with extension levels of pre-defined scale using the nine-point central difference were calculated. These calculations were performed for all the plotted load-extension curves. The data of slopes of each interval were analyzed for significance in differences, using a one-way ANOVA test at the 95% level of confidence. Therefore, the Tukey test was performed to categorize the homogeneous subsets. In the theoretical parts, a mechanical model was

TABLE III
CHARACTERISTICS OF THE FABRICS FROM COLLECTED DATA [20]

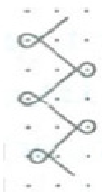
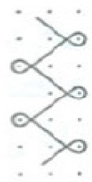
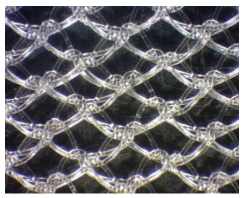
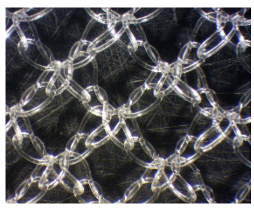
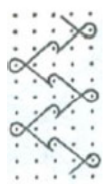
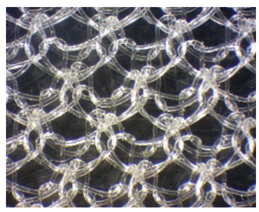
Fabric	Lapping movement		CPC (cm^{-1})	WPC (cm^{-1})	Weight (g/m^2)	Figure of fabrics
	Front guide bar	Back guide bar				
Pinhole net			10	6.2	129	
Sandfly			5.2	5.7	104	
Quasi-sandfly			5.2	6.1	148	

TABLE IV
SECOND ELASTIC MODULUS OBTAINED FROM EXPERIMENTAL INVESTIGATIONS AND THEORETICAL STUDIES FOR NWKF

Fabrics	Test direction	Second elastic modulus (N/mm)		Error (%)
		experimental	theoretical	
Produced fabrics	Course	1.192	1.127	5.453
	Wale	2.145	2.002	6.667
Pinhole-net*	Course	17.169	13.951	18.743
	Wale	12.404	9.514	23.299
Sandfly*	Course	4.775	4.003	16.166
	Wale	10.908	9.143	16.181
Quasi-sandfly*	Course	13.198	9.639	26.966
	Wale	13.466	10.471	22.126

*The experimental data were collected from references 15 and 16.

proposed based on the geometry of fabrics after jamming of the yarns using Energy method and Castigliano theorem. The proposed model was used for the calculation of the second elastic modulus of the fabrics in the course and wale directions.

Since the basic parameters of the fabrics were used to propose the model, it seems that the model can be applied to the other kinds of net fabrics to some extent. The collected data in the form of second elastic modulus, with the authors' own test data, were used to validate the proposed model. The calculated and measured second elastic modulus of the fabrics in the wale and course directions are given in Table IV. As can be seen from the table and also the calculated error, not only the model can predict the modulus of the authors' produced fabric with high accuracy, but also it can predict the modulus of other fabrics reasonably. It is worth mentioning that the errors for the authors' data were minor, however, the errors of prediction for the collected data were further. Furthermore, the calculated errors for sandfly fabrics were smaller than the other collected data. It refers to the structure of the sandfly fabrics that is analogous to the produced fabrics by the authors. The other reason is that the procedure followed for the development of the model was based on the shape of the holes and there is a marked difference between the configurations of the holes of the used fabrics. However, the proposed model could predict the modulus reasonably.

It should be noted that the modulus was obtained through the load-extension diagrams instead of the stress-strain diagrams owing to the point that the cross-sections of fabrics contain an unknown amount of space and thus the cross-sectional area is not clearly defined [22].

The most important result is that the second elastic modulus of the fabrics is a function of the fabric structure. This finding was rarely found in literature owing to the two reasons. First, there is not enough research into the second elastic behavior of the knitted fabrics. Second, most papers relate the second elastic behavior of the fabrics to the yarns mechanical behavior after the jamming status, and the effects of the configuration of yarns in the structure of the fabrics had been ignored. However, the proposed model indicated that the second elastic modulus is not independent of the fabric structure and stitch density, and the size of holes and the length of sides of holes influence the fabric behavior. It is important to note that the effectiveness of the fabric structural parameters is different for wale-wise and course-wise calculations.

IV. CONCLUSION

This study was devoted to investigate the tensile behavior of net warp-knitted fabrics after jamming of the constituent yarns. In other words, the second elastic behavior of fabrics in the course and wale directions was studied experimentally and theoretically. It is important to note that the second elastic modulus is important in the field of composite materials reinforced with the fabrics. Net-fabrics were used due to the lack of enough attention. A theoretical model was proposed based on the consideration of the geometry of fabrics after the jamming of the yarns. Energy method and Castigliano theorem were used to build the model. To assess the proposed model, uniaxial tensile tests in the wale and course directions were conducted. Furthermore, the experimental data from the other researches were used for validation of the proposed model. The results indicated that the mechanical model can predict the second elastic modulus sufficiently. This work showed that not only the second elastic modulus of knitted fabrics is the function of yarn modulus but also the structure of fabrics like stitch density, the size of holes, and the length of sides of holes affect the modulus considerably. This result was true despite the heretofore accepted concept.

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REFERENCES

[1] A. Kurbak, “Geometrical models for weft-knitted spacer fabrics”, *Text. Res. J.*, vol. 87, no. 4, pp. 409-423, 2017.

[2] M. Sheikhzadeh, M. Ghane, Z. Eslamian, and E. Pirzadeh, “A modeling study on the lateral compressive behavior of spacer fabrics”, *J. Text. Inst.*, vol. 101, no. 9, pp. 795-800, 2010.

[3] O. Kyzymchuk, L. Melnyk, and S. Arabuli, “Study of elastic warp knitted bands: production and properties”, *Tekstilec*, vol. 63, no. 2, pp. 113-123, 2020.

[4] K.H. Leong, S. Ramakrishna, Z.M. Huang, and G.A. Bibo, “The potential of knitting for engineering composites-a review”, *Compos. Part A: Appl. Sci. Manuf.*, vol. 31, no. 3, pp. 197-220, 2000.

[5] J.T. Oden and E.A. Ripperger, “Mechanics of elastic structures”, *J. Mech. Des.*, vol. 104, no. 4, pp. 681-682, 1981.

[6] H. Dabiryan and A.A.A. Jeddi, “Analysis of warp-knitted fabric structure. Part I: a 3D straight line model for warp knitted fabrics”, *J. Text. Inst.*, vol. 102, no. 12, pp. 1065-1074, 2011.

[7] H. Dabiryan and A.A.A. Jeddi, “Analysis of warp-knitted fabric structure. Part IV: modeling the initial Poisson’s ratio in elastic deformation region”, *J. Text. Inst.*, vol. 103, no. 12, pp. 1352-1360, 2012.

[8] H. Dabiryan, A.A.A. Jeddi, and A. Rastgoo, “Analysis of warp-knitted fabric structure. Part II: theoretical study on initial modulus of warp-knitted fabrics (tricot, locknit, and satin)”, *J. Text. Inst.*, vol. 103, no. 9, pp. 997-1011, 2012.

[9] H. Dabiryan, A.A.A. Jeddi, and A. Rastgoo, “Analysis of warp-knitted fabric structure. Part III: theoretical study on initial modulus of warp-knitted fabrics (reverse locknit, three-and four-needle sharkskin)”, *J. Text. Inst.*, vol. 103, no. 11, pp. 1213-1227, 2012.

[10] P. Grosberg, “The geometry of warp-knitted fabrics”, *J. Text. Inst. Proc.*, vol. 51, no. 1, pp. P15-P15, 1960.

[11] Y. Yanagawa, S. Kawabata, K. Nakagawa, K. Toyama, and N. Kawai, “Theoretical study on the biaxial tensile properties of single tricot warp knitted fabric with open lap”, *J. Text. Mach. Soc. Jpn.*, vol. 16, no. 6, pp. 216-228, 1970.

[12] Y. Kyosev, *Warp Knitted Fabrics Construction*, CRC, 2019.

[13] Y. Liu and H. Hu, “Compressive mechanics of warp-knitted spacer fabrics. Part I: a constitutive model”, *Text. Res. J.*, vol. 86, no. 1, pp. 3-12, 2016.

[14] Y. Liu and H. Hu, “Compressive mechanics of warp-knitted spacer fabrics”, *J. Ind. Text.*, pp. 1-21, 2019. doi: 10.1177/1528083719886054

[15] J. Yip and S.-P. Ng, “Study of three-dimensional spacer fabrics: physical and mechanical properties”, *J. Mater. Proc. Technol.*, vol. 206, no. 1-3, pp. 359-364, 2008.

[16] W. Renkens and Y. Kyosev, “Geometry modelling of warp knitted fabrics with 3D form”, *Text. Res. J.*, vol. 81, no. 4, pp. 437-443, 2011.

[17] Y. Kyosev, *Warp Knitting Pattern Editor 3D*, TexMind, Mönchengladbach, 2019.

[18] V. Ghorbani, A.A.A. Jeddi, and H. Dabiryan, “Theoretical and experimental investigation of tensile properties of net warp-knitted spacer fabrics”, *J. Text. Inst.*, vol. 111, no. 4, pp. 518-528, 2020.

[19] M. Badin Dahesh, “Experimental analysis of the relation between tensile and bursting properties of net warp knitted fabrics used as surgical mesh (in Persian)”, *M.Sc. thesis*, Department of Textile Engineering, Amirkabir University of Technology, Tehran, Iran, 2019.

[20] M. Badin Dahesh, A. Asayesh, and A.A.A. Jeddi, “The effect of fabric structure on the bursting characteristics of warp-knitted surgical mesh”, *J. Text. Inst.*, vol. 111, no. 9, pp. 1364-1353, 2020.

[21] K. Hosseini, A. Sadeghi, and A.A.A. Jeddi, “Characterization of fabric tensile loading curve in nonlinear region related to their structure; Part I: woven fabric”, *J. Text. Polym.*, vol. 1, no. 2, pp. 53-59, 2013.

[22] B. Saville, *Physical Testing of Textiles,* Cambridge: Woodhead, 1999.