Two Robust Fuzzy Regression Models and Their Applications in Predicting Imperfections of Cotton Yarn

Jalal Chachi, S. Mahmoud Taheri, Saeed Fattahi, and S. Abdolkarim Hosseini Ravandi

***Abstract*—Using the generalized Hausdorff-metric, two least-absolutes (LA) approaches to multiple fuzzy regression modeling are introduced for the case of crisp input-fuzzy output data. The main advantage of the proposed models is that they are not so sensitive to the outlier data points. The proposed models as well as two common fuzzy least-squares (LS) models are employed in a case study to estimate imperfections of cotton yarn using fiber properties in a real-life data. In order to derive the fuzzy regression models between imperfections of cotton yarn and fiber properties, first, effective variables are selected by the statistical stepwise test. Then, four fuzzy models, including two new LA models and two LS models, are sought to fit the data set.**

Finally, two criteria are employed to evaluate the goodness-of-fit of models. Moreover, a predictive ability index is introduced and employed to evaluate the predictability of the models. Using these criteria, a comparative study between the proposed fuzzy least-absolutes regression models and fuzzy least-squares regression models has also been addressed. The comparison results reveal that the LA-fuzzy models perform better than the LS-fuzzy models in imperfections of cotton yarn estimation for the particular data set used in this study.

***Keywords*: cross-validation, fuzzy least-absolutes regression, fuzzy least-squares regression, outlier, yarn quality properties**

I. Introduction

Modeling OF yarn and fiber properties have been popular topics in the field of textile engineering in recent decades. Especially, the main aim of many textile studies has been to predict important characteristics such as tensile, unevenness, and hairiness of yarn from fiber properties (see, e.g. [1-8]). The most popular tool to deal with such problems is the statistical regression analysis. Generally, in statistical regression, we can make estimates and predictions for a dependent (response) variable, based on a set of observed data of independent (predictor) variables. But, the in systems in which human intelligence plays a part, we usually encounter the following three problems:

- The observation of variables are imprecise (fuzzy) rather than crisp
- The relationship between variables is imprecise
- The sample size may be small due to practical limitations, so that we cannot justify the underlying basic assumptions for the basic statistical model.

To deal with such cases, we need, therefore, to develop some new soft procedures, especially fuzzy regression. Fuzzy regression methods, in a general perspective, include three categories:

I) The class of possibilistic methods: In these methods, which are based on the ideas proposed by Tanaka *et al*. [9, 10] using the possibilistic concepts, the fuzzy regression problem is formulated as a mathematical programming problem. This approach was investigated and improved by several authors [2,16].

II) The class of least squares and least absolutes methods: In these methods, the parameters of the model are estimated based on a distance on the space of fuzzy numbers [5,17-25].

III) The class of heuristic methods: This class includes some novel methods or some techniques which combine the possibilistic, least squares, least absolutes methods, and some other methods in classical regression (such as robust techniques, MARS method, piece-wise regression, etc.) in order to provide a fuzzy linear regression model [18,19,26-31].

For more about statistical methods in fuzzy environments, see [32,33].

In recent years, various soft methods [34], especially the methods based on fuzzy regression models, have received much attention from researchers in textile engineering analysis. Regarding proposes of this paper, we will briefly review some soft methods on the topic of predicting yarn properties from fiber properties.

Ertugrul and Tus [2] investigated a fuzzy linear programming method and studied its application in a textile firm. Fattahi *et al*. [5] studied cotton yarn engineering via fuzzy least-squares regression. Tavanai *et al*. [8] to analyze the color yield in polyethylene terephthalate dyeing with statistical and fuzzy regression models. Jeng-Jong [35] proposed a genetic algorithm for searching weaving parameters for woven fabrics. Kuo *et al*. [36] obtained a mathematical model for calculating extruder screw speed, gear pump speed, and winder speed proposed through fuzzy set theory to improve the quality of as-spun fibers in a melt spinning system. Majumdar *et al*. [37] presented the application of a hybrid neuro-fuzzy system for the prediction of cotton yarn strength from HVI fiber properties. A competitive study was done by Nasiri *et al*. [38] introduced a genetic-fuzzy approach to model polyester dyeing. Lu *et al*. [39] proposed a human-machine measure integrated fuzzy multi-criteria group decision-

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making method. The reader is referred to the paper of Sztandera and Pastore [34] which is a useful text on soft computing methods applied in textile science.

In the present paper, we introduce a two-stage method to construct two robust fuzzy regression models for crisp input-fuzzy output data. In the first stage, using a robust method called M-estimation method, the effective variables are selected. Then, by a robust technique (based on the least-absolutes method) and a least-squares method, two fuzzy regression models are fitted to the data set. In this part, each approach will be followed by two models, one with spreads restricted in sign and the other with spreads unrestricted in sign. Three criteria are employed to evaluate the goodness-of-fit and the predictive ability of the obtained models. The results show that the fuzzy LA models predict better than the fuzzy LS models.

This paper is organized as follows. In Section II, the main problem is briefly stated. Two basic fuzzy regression methods are explained in Sections III and IV. Three criteria for evaluation of the models are presented in Section V. In Section VI, the method of data collection is presented. The results and discussions of statistical tests and fuzzy regression models are given in Sections VII and VIII. Finally, in Section IX the conclusion remarks are presented.

The mathematical background for statistical methods and fuzzy arithmetic are given in Appendices A, and B.

II. Statement of The Main Problem

Assume that, in a practical study, the observed data on n statistical units are as follows:

$$(\tilde{y}_1, \boldsymbol{x}_1), \dots, (\tilde{y}_n, \boldsymbol{x}_n),$$

where $\tilde{\boldsymbol{y}} = (\tilde{y}_1, \dots, \tilde{y}_n)^t$ is the vector of symmetric triangular fuzzy numbers, i.e. $y_i = (y_i, s_i)_T$ $(i = 1, \dots, n)$, which determines the fuzzy observation of the dependent variable, and

$$\boldsymbol{x_i} = [x_{0i}, x_{1i}, \dots, x_{ki}] \ (i = 1, \dots, n;\ k < n;\ x_{0i} = 1),$$

forms the vector of crisp observed independent variables. Without loss of generality, we can assume that $x_{ji} > 0$, by a simple translation of all data if necessary [25,40,41]. So, we will consider the following functional dependence between $\boldsymbol{y}_{n\times 1}$ and $\boldsymbol{X}_{n\times(k+1)}$

$$\tilde{\boldsymbol{y}}_{n\times 1} = \boldsymbol{X}_{n\times(k+1)} \tilde{\boldsymbol{\beta}}_{(k+1)\times 1} \quad (1)$$

$$\begin{bmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_1} \\ \vdots \\ \boldsymbol{x}_n \end{bmatrix} \begin{bmatrix} (\beta_0, \sigma_0)_T \\ \vdots \\ (\beta_k, \sigma_k)_T \end{bmatrix} = \begin{bmatrix} \left(\sum_{j=0}^k x_{j1}\beta_j, \sum_{j=0}^k x_{j1}\sigma_j\right)_T \\ \vdots \\ \left(\sum_{j=0}^k x_{jn}\beta_j, \sum_{j=0}^k x_{jn}\sigma_j\right)_T \end{bmatrix} \quad (2)$$

The procedure for estimating the fuzzy parameter $\tilde{\boldsymbol{\beta}}_{(k+1)\times 1}$ is based on choosing the best candidate $\hat{\tilde{\boldsymbol{\beta}}}_{(k+1)\times 1}$ instead of $\tilde{\boldsymbol{\beta}}_{(k+1)\times 1}$, consisting in minimizing the total difference between the observed values of the response variable, $\tilde{\boldsymbol{y}}_{n\times 1}$, and its theoretical counterpart, $\hat{\tilde{\boldsymbol{y}}}_{n\times 1}$, defined by

$$\hat{\tilde{\boldsymbol{y}}}_{n\times 1} = \boldsymbol{X}_{n\times(k+1)} \hat{\tilde{\boldsymbol{\beta}}}_{(k+1)\times 1} \quad (3)$$

with respect to a suitable distance [18].

In the sequel, for simplicity, we use $\boldsymbol{X}$ instead of $\boldsymbol{X}_{n\times(k+1)}$, $\tilde{\boldsymbol{y}}$ and $\hat{\tilde{\boldsymbol{y}}}$ instead of $\tilde{\boldsymbol{y}}_{n\times 1}$and $\hat{\tilde{\boldsymbol{y}}}_{n\times 1}$, and $\tilde{\boldsymbol{\beta}}$ and $\hat{\tilde{\boldsymbol{\beta}}}$ instead of $\tilde{\boldsymbol{\beta}}_{(k+1)\times 1}$ and $\hat{\tilde{\boldsymbol{\beta}}}_{(k+1)\times 1}$, respectively.

III. The Fuzzy Least-Squares Regression Model

In the following, we briefly illustrate a common least-squares method for estimating the fuzzy parameter $\tilde{\boldsymbol{\beta}}$. In the fuzzy least-squares method, using a distance between fuzzy numbers, the parameters of the model are estimated so that the total error would be minimized. A well-known distance between two symmetric triangular fuzzy numbers $\tilde{M} = (m, \gamma_m)_T$ and $\tilde{N} = (n, \gamma_n)_T$ is defined as follows [21, 22, 25]

$$d^2(\tilde{M}, \tilde{N}) = (n - m)^2 + \tfrac{1}{6}(\gamma_m - \gamma_n)^2 \quad (4)$$

For estimating the fuzzy parameter $\tilde{\boldsymbol{\beta}}$, we should minimize the sum of squared errors, i.e. $\sum_{i=1}^n d^2(\tilde{y}_i, \hat{\tilde{y}}_i)$. Using fuzzy arithmetic methods (see Appendix A) [42], such an estimation problem can be stated as the following optimization problem

$$\min_{\tilde{\boldsymbol{\beta}}} \begin{Bmatrix} \sum_{i=1}^n \left(y_i - \sum_{j=0}^k x_{ji}\beta_j\right)^2 + \\ \frac{1}{6}\sum_{i=1}^n \left(s_i - \sum_{j=0}^k x_{ji}\sigma_j\right)^2 \end{Bmatrix} \quad (5)$$

$$s.t.\ \sigma_j, \beta_j \in R,\ \ j = 0,1, \dots, k$$

Differentiating the objective function of the above optimization problem with respect to the coefficients σ_j and β_j, $j = 0,1, \dots, k$, and setting the partial derivatives to 0 leads to the following matrix forms

$$\boldsymbol{X}^t\boldsymbol{X}\,\boldsymbol{\beta} = \boldsymbol{X^t y} \quad (6)$$

$$\boldsymbol{X}^t\boldsymbol{X}\,\boldsymbol{\sigma} = \boldsymbol{X^t s} \quad (7)$$

where $\boldsymbol{\beta} = (\beta_0, \dots, \beta_k)$ and $\boldsymbol{\sigma} = (\sigma_0, \dots, \sigma_k)$ are the center values and the spread values of the fuzzy coefficient $\tilde{\boldsymbol{\beta}}$, respectively, $\boldsymbol{y} = (y_1, \dots, y_n)$ and $\boldsymbol{s} = (s_1, \dots, s_n)$ are the center values and the spread values of the fuzzy response $\tilde{\boldsymbol{y}}$, respectively.

It is shown that if $Rank(\boldsymbol{X}) = n + 1$, and $(\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X^t s} > \boldsymbol{0}$, then the least-squares optimization problem has unique solutions as follows [25]

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X^t y} \quad (8)$$

$$\hat{\boldsymbol{\sigma}} = (\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X^t s} \quad (9)$$

Remark 1: The above mentioned conditions guarantee that the estimated spreads of the fuzzy parameters, i.e. σ_j $j = 0,1, \dots, k$, will be non-negative. However, it is possible to encounter situations in which $(\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X^t s} < \boldsymbol{0}$. Under this circumstance, which was not considered by Xu and Li [25], the solutions are not guaranteed. To remove this difficulty, Mohammadi and Taheri [21] suggested a procedure which will be our reference method when $(\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X}^t\boldsymbol{s} < \boldsymbol{0}$ occurs.

Remark 2: According to Chang and Lee [12] we can drop the above conditions. In this manner, the spreads of some fuzzy coefficients for the optimal model may be obtained as negative amounts. Note that by using this method, zero values would be considered for negative predicted spreads of fuzzy response, i.e. $\hat{\boldsymbol{s}} = \max(\boldsymbol{0}, \boldsymbol{X}\hat{\boldsymbol{\sigma}})$ [29]. This method is called the fuzzy linear regression model with spreads unrestricted in signs (SUS) [12].

Regarding the above remarks, we will follow the LS method in two versions:

1) The LS method with spreads restricted in sign (Remark 1);
2) The LS method with spreads unrestricted in sign (LS-SUS) (Remark 2).

IV. The Proposed Fuzzy Least-Absolutes Regression Model

Although the LS approach is a popular method to estimate the unknown parameters, it is sensitive in terms of outlier data points [43,44]. In such cases, when some outliers exist in data set, it is usually preferred to use a robust method. In practice, regression modeling based on the least-absolutes method has been used as a robust method [20,24,40]. In this section, using the generalized Hausdorff-metric D_1 on the set of symmetric triangular fuzzy numbers (see Appendix B) [45,46], we introduce a novel least-absolutes method to estimate the parameter $\widetilde{\boldsymbol{\beta}}$.

In this case, the least-absolutes optimization problem can be stated as follows

$$\min_{\widetilde{\boldsymbol{\beta}}} D_1(\widetilde{\boldsymbol{y}}, \boldsymbol{X}\widetilde{\boldsymbol{\beta}}) \tag{10}$$

or equivalently

$$\min_{\widetilde{\boldsymbol{\beta}}} \begin{Bmatrix} \sum_{i=1}^{n}\left|y_i - \sum_{j=0}^{k} x_{ji}\beta_j\right| + \\ \frac{1}{2}\sum_{i=1}^{n}\left|s_i - \sum_{j=0}^{k} x_{ji}\sigma_j\right| \end{Bmatrix} \tag{11}$$

$$s.t.\, \sigma_j \in R^+, \beta_j \in R, \;\; j = 0,1,\dots,k$$

which is a constrained non-linear programming problem. The minimization of D_1 over $\widetilde{\boldsymbol{\beta}}$ can separately be solved: once for all possible candidates for $\boldsymbol{\beta}$, and then for all possible candidates for $\boldsymbol{\sigma}$, respectively. Thus, this optimization problem can be rewritten as the following two sub-optimization non-linear programming problems [47,48]

$$\min_{\boldsymbol{\beta}}\left\{\sum_{i=1}^{n}\left|y_i - \sum_{j=0}^{k} x_{ji}\beta_j\right|\right\} \tag{12}$$

$$s.t.\, \beta_j \in R, \qquad j = 0,1,\dots,k$$

$$\min_{\boldsymbol{\sigma}}\left\{\frac{1}{2}\sum_{i=1}^{n}\left|s_i - \sum_{j=0}^{k} x_{ji}\sigma_j\right|\right\} \tag{13}$$

$$s.t.\, \sigma_j \in R^+, \qquad j = 0,1,\dots,k$$

In order to simplify the above optimization problems, we show how, by introducing additional variables, two linear programming problems can handle the non-linear optimization problems ***a)*** and ***b)*** (for more details, see, e.g. [47,48]).

First, we consider the sub-optimization problem ***a)***. Let ε_i^+ and ε_i^-, $i = 1,\dots,n$, represent two non-negative variables so that

$$\left|y_i - \sum_{j=0}^{k} x_{ji}\beta_j\right| = \varepsilon_i^+ + \varepsilon_i^- \tag{14}$$

$$y_i - \sum_{j=0}^{k} x_{ji}\beta_j = \varepsilon_i^+ - \varepsilon_i^- \tag{15}$$

Let us consider the following matrix notations

$$\boldsymbol{\varepsilon}_{n\times1}^+ = (\varepsilon_1^+, \dots, \varepsilon_n^+)^t \tag{16}$$

$$\boldsymbol{\varepsilon}_{n\times1}^- = (\varepsilon_1^-, \dots, \varepsilon_n^-)^t \tag{17}$$

$$\boldsymbol{e}_{(k+1+2n)} = (\boldsymbol{\beta}^t \; (\boldsymbol{\varepsilon}_{n\times1}^+)^t \; (\boldsymbol{\varepsilon}_{n\times1}^-)^t)^t \tag{18}$$

$$\boldsymbol{H}_{\boldsymbol{n}\times(k+1+2n)} = (\boldsymbol{X} \; \boldsymbol{I}_{\boldsymbol{n}\times\boldsymbol{n}} \; -\boldsymbol{I}_{\boldsymbol{n}\times\boldsymbol{n}}) \tag{19}$$

$$\boldsymbol{h}_{(k+1+2n)\times1} = \left(\boldsymbol{0}_{1\times(k+1)} \; \boldsymbol{J}_{1\times2n}\right)^t \tag{20}$$

where $\boldsymbol{I}_{\boldsymbol{n}\times\boldsymbol{n}}$ is an identity matrix of order n, and $\boldsymbol{J}_{1\times2n}$ denotes the $1\times2n$-vector of 1's. Now, the non-linear optimization problem **a)** becomes equivalent to the following linear optimization problem

$$\min_{\boldsymbol{e}_{(k+1+2n)}} \left(\boldsymbol{h}_{(k+1+2n)\times1}\right)^t \boldsymbol{e}_{(k+1+2n)} \tag{21}$$

$$s.t.\; \boldsymbol{H}_{\boldsymbol{n}\times(k+1+2n)}\boldsymbol{e}_{(k+1+2n)} = \boldsymbol{y}_{n\times1} \tag{22}$$

$$\boldsymbol{\varepsilon}_{n\times1}^+ \in R^{+n}, \boldsymbol{\varepsilon}_{n\times1}^- \in R^{+n}, \boldsymbol{\beta} \in R^{k+1}$$

This problem can be solved by common software program. In the present work, we used MATLAB software [49] for numerical studies concerning the above optimization problem.

The same method may easily be used to solve the optimization problem **b)**. In this case, we replace $\boldsymbol{\beta} \in R^{k+1}$ with $\boldsymbol{\sigma} \in R^{+(k+1)}$, i.e. all variables are assumed to be nonnegative.

Remark 3: Concerning Remark 2, in fuzzy LA method, we can also drop the condition $\boldsymbol{\sigma} \in R^{+(k+1)}$ in the optimization problem. So, in this paper, we will follow the LA method in two versions:

1) *The LA method with spreads restricted in sign, i.e. considering the constraint $\boldsymbol{\sigma} \in R^{+(k+1)}$ in the optimization problem*
2) *The LA method with spreads unrestricted in sign (LA-SUS), i.e. considering the constraint $\boldsymbol{\sigma} \in R^{(k+1)}$ in the optimization problem. Using the LA-SUS method, zero values would be considered for negative predicted spreads of fuzzy response, i.e. $\hat{\boldsymbol{s}} = \max(\boldsymbol{0}, \boldsymbol{X}\hat{\boldsymbol{\sigma}})$.*

V. Methods of Evaluation of The Model

In this study, we use three well-known criteria to evaluate the obtained fuzzy regression models. The first and the second ones are common indices for evaluating the goodness-of-fit of the fuzzy regression models used by many authors [16-29,40]. The third one is proposed to evaluate the predictability of the fuzzy regression models.

A. Goodness-of-Fit Indices

I) Mean Relative Error (MRE): This criteria, which was introduced by Kim and Bishu [29], is defined as

$$MRE = \frac{1}{n}\sum_{i=1}^{n} E\left(\tilde{y}_i, \hat{\tilde{y}}_i\right) \quad (23)$$

where

$$E\left(\tilde{y}_i, \hat{\tilde{y}}_i\right) = \int \frac{\left|\tilde{y}_i(x) - \hat{\tilde{y}}_i(x)\right|}{\int \tilde{y}_i(x)} dx \quad (24)$$

This index is the ratio of the total difference between the estimated and observed membership values of response variable to the total observed membership values of the response variable.

II) Mean Similarity Measure (MSM): This index is defined based on the similarity of fuzzy numbers [28,30] as

$$MSM = \frac{1}{n}\sum_{i=1}^{n} S\left(\tilde{y}_i, \hat{\tilde{y}}_i\right) \quad (25)$$

in which

$$S\left(\tilde{y}_i, \hat{\tilde{y}}_i\right) = \frac{\int \min\left(\tilde{y}_i(x), \hat{\tilde{y}}_i(x)\right) dx}{\int \max\left(\tilde{y}_i(x), \hat{\tilde{y}}_i(x)\right) dx} \quad (26)$$

TABLE I
A SUMMARY OF THE STATISTICAL RESULTS FOR FIBER AND YARN IMPERFECTIONS

Properties	min	Max	Mean	S.D.
X_1: Fiber length (UHML)-(mm)	28.89	30.30	29.67	0.50
X_2: Mean length (ML)-(mm)	24.51	26.21	25.31	0.52
X_3: Uniformity index (U.I)-(%)	84.00	86.50	85.28	0.81
X_4: Fiber bundle tenacity (gr/tex)	29.90	38.60	33.85	2.28
X_5: Fiber elongation (%)	7.10	7.20	7.11	0.03
X_6: Short fiber index (S.F.I)-(%)	3.00	5.30	3.79	0.64
X_7: Micronaire (gr/in)	3.90	4.75	4.31	0.20
X_8: Roving unevenness (CV%)	3.80	13.80	8.06	2.70
X_9: Roving count (Ne)	0.89	1.42	1.11	0.19
X_{10}: Yarn count (Ne)	15.71	32.16	23.18	5.23
X_{11}: Fiber maturity (M.R)-(%)	83.00	86.00	84.42	0.75
Y: Yarn imperfection (n/1000m)	20.00	1110	488.32	291.7

B. Predictive Ability Index

According to the Cross-Validation method [41], and to evaluate the predictability of the fuzzy regression models, we introduce and employ the concept of Mean Predictive Ability (MPA), as follows

$$MPA = \frac{1}{n}\sum_{i=1}^{n} E\left(\tilde{y}_i, \hat{\tilde{y}}^{i}\right) \quad (27)$$

in which $\hat{\tilde{y}}^{i}$ is the predicted value of the dependent variable with the fuzzy regression model for which the ith observation is left out from the data set while the remaining observations are used to develop the model.

VI. DATA COLLECTION

In this study, a total of twelve different rovings obtained from carded cotton were selected. The atmospheric conditions of spinning mills were standard. The spinning operations can affect the fiber properties in different ways, depending on the machinery line, and the adjustments among others. Thus, in order to minimize the random errors and eliminate these effects, fiber properties were measured from rovings (50 to 60 grams of rovings were untwisted carefully). Twelve cotton fiber properties (Fiber length (UHML)-(mm), Mean length (ML)-(mm), Uniformity index (U.I)-(%), Fiber bundle tenacity (gr/tex), Fiber elongation (%), Short fiber index (S.F.I)-(%), Micronaire (gr/in), Roving unevenness (CV%), Roving count (Ne), Yarn count (Ne), Fiber maturity (M.R)-(%), Yarn imperfection (n/1000m)) were measured by Premier HVI testing system (HFT 9000 V2). All samples (40 Yarn samples) were spun into yarns on a SKF lap spinner machine under standard conditions at yarn counts of 16, 20, 24, 28, and 32 Ne (2 or 3 yarn counts from each roving according to roving count). Each yarn count was spun at optimum twist factor. The appropriate settings were adjusted on the ring spinning machine for each sample. Other spinning conditions were kept constant. The unevenness test of yarns and rovings were measured on the premier evenness tester (7000 V3). The values of the main properties of these variables are shown in Table I.

VII. STATISTICAL TESTS FOR SELECTING APPROPRIATE VARIABLES

We used the stepwise test for selecting suitable variables. The stepwise selection is a modification of forward selection in which at each step all regressors previously entered into the model are reassessed via their partial F-statistics. A regressor added at an earlier step may now be redundant because of the relationships between it and regressors now in the model. If the partial F-statistic for a variable is less than F_{out}, that variable is dropped from the model [41]. Table II presents the summary of stepwise selection of independent variables. Based on this table, the stepwise selection terminates with

TABLE II
A SUMMARY OF STEPWISE SELECTION OF INDEPENDENT VARIABLES

Step	Entered variable	Removed variable	No. of variables in the model	Partial R-square	Model R-square	F-value	p-value
1	X_{10}	--	1	0.55	0.55	47.27	0.0001
2	X_8	--	2	0.13	0.68	14.65	0.0005
3	X_4	--	3	0.05	0.73	7.44	0.0098
4	X_7	--	4	0.02	0.75	2.64	0.0113

four variables having the highest influence on the yarn imperfections (Y) including the fiber bundle tenacity (X_4), micronaire (X_7), roving unevenness (X_8), and yarn count (X_{10}). Now, we recall the obtained effective independent variables as

- x_1: fiber bundle tenacity
- x_2: micronaire
- x_3: roving unevenness
- x_4: yarn count

The recorded data for such variables are presented in Table III.

TABLE III
THE DATA SET INCLUDING CRISP OBSERVATIONS OF THE EXPLANATORY VARIABLES (FIBER BUNDLE TENACITY (x_1), MICRONAIR (x_2), ROVING UNEVENNESS (x_3), AND YARN COUNT (x_4)) AND THE FUZZY RESPONSE OBSERVATIONS (YARN IMPERFECTIONS $(y,s)_T$)

No.	x_1	x_2	x_3	x_4	$(y,s)_T$
1	32.5	4.45	6.15	15.92	$(245,24)_T$
2	32.5	4.45	6.15	19.71	$(375,37)_T$
3	32.5	4.45	6.15	23.78	$(510,51)_T$
4	35.5	4.40	9.00	15.71	$(495,49)_T$
5	35.5	4.40	9.00	19.52	$(760,38)_T$
6	35.9	4.45	4.90	15.89	$(105,10)_T$
7	35.9	4.45	4.90	19.99	$(180,18)_T$
8	35.9	4.45	4.90	24.07	$(280,28)_T$
9	32.4	4.20	9.70	24.26	$(465,46)_T$
10	32.4	4.20	9.70	28.81	$(595,59)_T$
11	32.4	4.20	9.70	32.16	$(1555,77)_T$
12	34.2	3.90	8.90	20.05	$(230,23)_T$
13	34.2	3.90	8.90	24.13	$(435,43)_T$
14	34.2	3.90	8.90	28.59	$(910,45)_T$
15	35.6	4.35	5.00	15.72	$(163,16)_T$
16	35.6	4.35	5.00	20.05	$(235,23)_T$
17	35.6	4.35	5.00	24.07	$(275,27)_T$
18	29.9	4.30	9.10	28.34	$(715,35)_T$
19	29.9	4.30	9.10	32.14	$(995,49)_T$
20	38.6	4.75	3.80	15.88	$(30,3)_T$
21	38.6	4.75	3.80	19.78	$(20,2)_T$
22	38.6	4.75	3.80	23.80	$(20,2)_T$
23	34.0	4.35	13.80	20.43	$(365,36)_T$
24	34.0	4.35	13.80	24.40	$(565,56)_T$
25	34.0	4.35	13.80	27.82	$(805,40)_T$
26	35.7	4.20	8.50	27.11	$(610,61)_T$
27	35.7	4.20	8.50	31.15	$(850,42)_T$
28	31.6	4.40	8.50	23.37	$(565,56)_T$
29	31.6	4.40	8.50	28.63	$(405,40)_T$
30	31.6	4.40	8.50	31.96	$(815,40)_T$
31	33.3	4.10	11.70	24.10	$(415,41)_T$
32	33.3	4.10	11.70	27.32	$(855,42)_T$
33	33.3	4.10	11.70	31.65	$(1110,55)_T$
34	30.2	4.10	7.60	16.60	$(260,26)_T$
35	30.2	4.10	7.60	20.69	$(440,44)_T$
36	30.2	4.10	7.60	24.40	$(775,38)_T$
37	34.4	4.30	6.70	15.72	$(400,40)_T$
38	34.4	4.30	6.70	19.72	$(425,42)_T$
39	34.4	4.30	6.70	23.76	$(575,57)_T$
40	33.9	4.40	9.10	16.11	$(145,14)_T$

On the other hand, it seems that there are some outliers in this data. Therefore, we used the *M*-estimation method for detecting outliers and providing stable results in the presence of outliers [43]. Table IV displays outlier point diagnostics.

Standardized robust residuals are computed based on the estimated parameters. Both the Mahalanobis distance and the robust MCD (Minimum Covariance Determinant) distance are displayed in Table IV. Outliers are defined by the standardized robust residuals and robust MCD distances which exceed the corresponding cutoff value (Cutoff=3). Twelve observations displayed in Table IV are outliers because their standardized robust residuals exceed the cutoff value in absolute value. It is remarkable that, we used the software SAS [50] for the above statistical analysis.

TABLE IV
THE LIST OF OUTLIER OBSERVATIONS

No. of outliers	Mahalanobis distance	Robust MCD distance	Standardized robust residual
4	3.27	0.00	9.50
5	2.99	0.00	11.04
11	3.44	0.00	4.17
12	3.21	0.59	-6.83
13	2.77	0.59	-5.19
21	2.98	0.64	-5.06
22	3.30	0.64	-4.52
28	3.56	0.72	3.53
29	3.14	0.72	-4.70
31	3.27	1.05	-5.52
36	3.16	0.65	3.10
37	2.03	0.14	4.14

TABLE V
COMPARISON BETWEEN LA AND LS MODELS (THE MODELS WITH SPREADS RESTRICTED IN SIGN)

Models	MSM	MRE	MPA
LA Model	0.1084	2.4281	2.7424
LS Model	0.0653	3.3881	3.4208

VIII. RESULTS AND DISCUSSIONS

A. *Optimal Models with Spreads Restricted in Sign*

By employing the LS method (described in Section III) to the data set given in Table III, the optimal model is obtained as follows

$$\hat{\tilde{y}}_{LS} = (847.12,0)_T + (-18.71,0)_T x_1 + (-179.64,0)_T x_2 + (24.24,2.63)_T x_3 + (37.26,1.34)_T x_4 \quad (28)$$

Moreover, using the LA method (described in Section IV) the optimal model is obtained as follows

$$\hat{\tilde{y}}_{LA} = (313.99,0)_T + (-20.29,0)_T x_1 + (-39.25,0)_T x_2 + (7.30,3.04)_T x_3 + (41.43,0.54)_T x_4 \quad (29)$$

A comparison between these models is done based on the goodness-of-fit criteria as well as the mean predictive ability index. The results are summarized in Table V.

B. *Optimal Models with Spreads Unrestricted in Sign*

By employing the LS-SUS method to the data set given in Table III, the optimal model is obtained as follows

$$\hat{\tilde{y}}_{LS-SUS} = (847.12,109.18)_T + (-18.71,-1.57)_T x_1 + (-179.64,-13.82)_T x_2 + (24.24,1.72)_T x_3 + (37.26,1.13)_T x_4 \quad (30)$$

In addition, the LA-SUS method yields the following optimal model

$$\hat{\tilde{y}}_{LA-SUS} = (313.99,154.29)_T + (-20.29,-2.05)_T x_1 + (-39.25,-19.59)_T x_2 + (7.30,1.09)_T x_3 + (41.43,1.06)_T x_4 \quad (31)$$

TABLE VI
COMPARISON BETWEEN LA-SUS AND LS-SUS MODELS (THE MODELS WITH SPREADS UNRESTRICTED IN SIGN)

Models	MSM	MRE	MPA
LA-SUS Model	0.1272	1.8555	2.2192
LS-SUS Model	0.0552	2.2495	2.3883

The results of comparison between these models are given in Table VI. As it is shown in Tables V and VI, the LA method provides models with more efficiency than the LS models, with both spreads restricted in sign and spreads unrestricted in sign. Moreover, except the MSM for the LS method, the models with spreads unrestricted in sign have higher goodness-of-fit indices as well as higher predictive ability index than those of the models with spreads restricted in sign.

C. Prediction of a New Case

The above models can also be adopted for forecasting the amount of response variable for new collected values of explanatory variables. For example, suppose that for a new sample we observe $(x_1,x_2,x_3,x_4) = (33,4.50,12,21)$ for the explanatory variables and we want to forecast the amount of the response variable. By substituting these values into the models with spreads restricted in sign, the predicted responses based on the LS model and the LA model are obtained as follows

$$\hat{\tilde{y}}_{LS} = (494.65,59.70)_T \quad (32)$$

$$\hat{\tilde{y}}_{LA} = (425.43,47.82)_T \quad (33)$$

So, according to the LS model, the predicted value of yarn imperfection is $(494.65,59.70)_T$. It means that the value of yarn imperfection is about 494.65 with a spread value of 59.70. According to the LA model, the predicted value of yarn imperfection is $(425.43,47.82)_T$, i.e. it would be about 425.43 with a spread value of 47.82. The membership functions of such fuzzy numbers are shown in Fig. 1.

Similarly, by substituting the new value $(x_1,x_2,x_3,x_4) = (33,4.50,12,21)$ into the models with spreads unrestricted in sign, the predicted responses based on the LS-SUS model and
the LA-SUS model are obtained as follows:

$$(\hat{y}_{LS-SUS}, \hat{s}_{LS-SUS})_T = (494.65,39.55)_T \quad (34)$$

$$(\hat{y}_{LA-SUS}, \hat{s}_{LA-SUS})_T = (425.43,33.83)_T \quad (35)$$

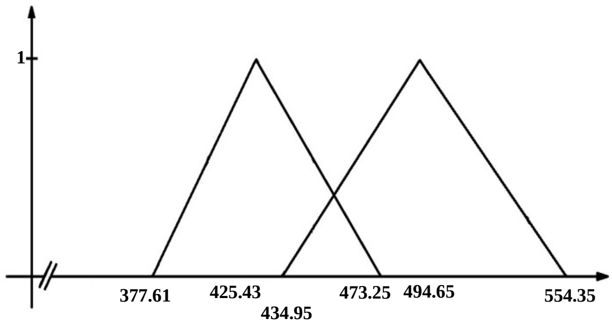

Fig. 1. Predicted values based on the optimal models with spreads restricted in sign.

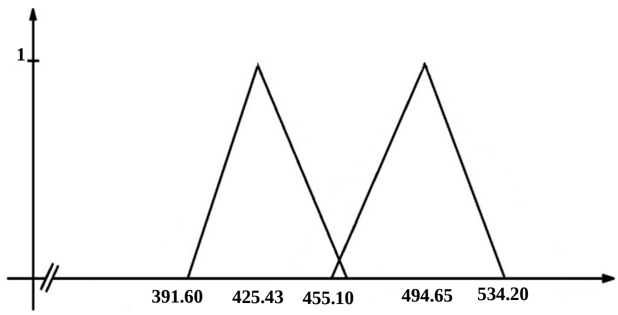

Fig. 2. Predicted values based on the optimal models with spreads unrestricted in sign.

Therefore, according to the LS-SUS model the predicted value of yarn imperfection is $(494.65,39.55)_T$, i.e. it is about 494.65 with a spread value of 39.55, and according to the LA-SUS model the predicted value of yarn imperfection is $(425.43,33.83)_T$, i.e. it is about 425.43 with a spread value of 33.83. The membership functions of the predicted values are shown in Fig. 2.

It is remarkable that, the prediction of yarn imperfection based on the LA and LA-SUS models has more precision than those of the LS and LS-SUS models. Note that the vagueness (imprecision) in prediction by LA and LA-SUS models are 47.82 and 33.83, respectively, while the vagueness (imprecision) in prediction by LS and LS-SUS models are 59.70 and 39.55, respectively.

IX. CONCLUSION AND REMARKS

In this paper, we introduced some novel models, with minimum random errors and maximum accuracy, to predict the imperfections of ring spun cotton yarns using fiber properties. In this regard, the following fuzzy regression models:

1) Least squares fuzzy regression model with spreads unrestricted in sign; and

2) Least absolutes fuzzy regression model with spreads unrestricted in sign,

are originally defined and applied for the first time in the field of textile engineering. The unrestricted in sing idea for estimating fuzzy parameters of the models was first introduced by Chang and Lee [12]. So, by the inception of this work we defined the above two new original fuzzy regression models. We also defined the least absolutes fuzzy regression model with spreads restricted in sign based on the definition of the generalized Hausdorff-metric between fuzzy numbers (see also [40]).

The application of this method is given for the first time in the present paper in textile engineering. Finally, in order to provide a competitive study the proposed fuzzy regression models are compared with the existing least squares fuzzy regression model proposed by Xu and Li [25]. By the results of the applied numerical example we concluded which kind of the fuzzy regression models used in this application is the best. Note that this conclusion is based on the data set used in this application.

It is remarkable that generally talking about the performance of any fuzzy regression model needs a variety of different examples. But the data sets used in the examples given in this paper have been collected in the real world providing the competitive study more strong. To this end, we used untwisted rovings for measuring fiber properties. We produced the cotton yarns with optimum twist factor due to the maximum performance of length and fineness of fibers in yarn. Because of some limitations in measurements, the observed data and/or the relationships among variables might not be considered as crisp quantities. Thus, commonly used LS fuzzy multiple linear regression models were adapted for the estimation of yarn imperfections. Concerning some outliers in data set, two novel LA fuzzy multiple linear regression models were also used for such estimations. The results indicate that the proposed robust methods are able to determine the regression coefficients with better explanatory powers. Also, yarn imperfections are influenced by fiber tenacity, micronaire, roving unevenness, and yarn count. Finally, four optimal models indicate that increasing roving unevenness, and yarn count, will increase the yarn imperfections (positive effect). Moreover, increasing fiber tenacity and fiber micronaire will reduce the yarn imperfections (negative effect).

The proposed robust approaches can be extended to other cases in textile engineering where the sample size is small and/or the data are non-precise (fuzzy), and/or there are some outliers in the data set. Furthermore, the study of combined models, using the robust fuzzy regression and neural networks for the case of huge sample size (including outliers) is a potential subject for further research.

X. Appendix A: Elementary Fuzzy Arithmetic

A fuzzy set $\tilde{A}$ on the universal set X is described by its membership function $\tilde{A}(x): X \to [0,1]$. In this paper, we assume that $X = R$, the set of real numbers. The crisp set $A_\alpha = \{x \in R | \tilde{A}(x) \geq \alpha\}$, $\alpha \in (0,1]$ is called the α-cut of $\tilde{A}$, and for $\alpha = 0$ we assume $A_0 = cl\{x \in R | \tilde{A}(x) > 0\}$, where cl is the closure operator. A fuzzy set of R whose α-cuts are non-empty closed intervals, for each $\alpha \in [0,1]$ is called a fuzzy number.

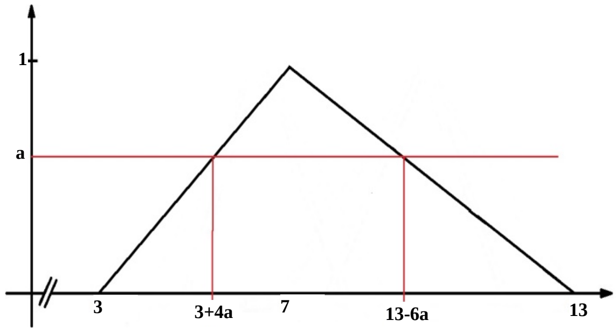

Fig. 3. The membership function and the α-cut of triangular fuzzy number $\tilde{N} = (7,4,6)_T$.

A specific class of fuzzy numbers on R, which is rich and flexible enough to cover most of the applications, is the so-called triangular fuzzy numbers $\tilde{N} = (n, l, r)_T$ with central value $n \in R$, and left and right spreads $l \in R^+$ and r$\in R^+$. The membership function and the α-cut of the triangular fuzzy number $\tilde{N}$ are as follows

$$\tilde{N}(x) = \begin{cases} \frac{x-(n-l)}{l}, & x \in [n-l, n] \\ \frac{(n+r)-x}{r}, & x \in [n, n+r] \end{cases} \quad (36)$$

$$N_\alpha = [n - (1-\alpha)l, n + (1-\alpha)r] = [N_\alpha^l, N_\alpha^r] \quad (37)$$

$$\alpha \in [0,1]$$

For $l = r$ the triangular fuzzy number $\tilde{N}$ is called symmetric and is abbreviated by $\tilde{N} = (n, l)_T$. As an example, the membership function and the α-cut of triangular fuzzy number $\tilde{N} = (7,4,6)_T$, which are as follows,

$$\tilde{N}(x) = \begin{cases} \frac{x-3}{4}, & x \in [3,7] \\ \frac{13-x}{6}, & x \in [7,13] \end{cases} \quad (38)$$

$$N_\alpha = [7 - 4(1-\alpha), 7 + 6(1-\alpha)] \quad (39)$$

$$= [3 + 4\alpha, 13 - 6\alpha]$$

$$\alpha \in [0,1]$$

are depicted in Fig. 3.

The algebraic operations of fuzzy numbers have been developed on the basis of Zadeh's extension principle [42]. Specially, if $\tilde{M} = (m, l_m, r_m)_T$ and $\tilde{N} = (n, l_n, r_n)_T$ are two triangular fuzzy numbers and γ is a real number, then

$$\gamma\tilde{M} = \begin{cases} (\gamma m, \gamma l_m, \gamma r_m) & \gamma > 0 \\ I_{\{0\}} & \gamma = 0 \\ (\gamma m, |\gamma| r_m, |\gamma| l_m) & \gamma < 0 \end{cases} \quad (40)$$

$$\tilde{M} + \tilde{N} = (m + n, l_m + l_n, r_m + r_n)_T \quad (41)$$

where I_A stands the characteristic function of a crisp set A.

XI. Appendix B: The Hausdorff-Metric and its Generalization

Let $K_c(R)$ be the family of all non-empty compact convex sets on R. The Hausdorff-metric between sets $A, B \in K_c(R)$ is defined as follows [45], [46]

$$d_H(A,B) = \max\{\sup_{b\in B}\inf_{a\in A}|a-b|, \sup_{a\in A}\inf_{b\in B}|a-b|\} \quad (42)$$

In special case, if $I_1 = [a_1, a_2]$ and $I_2 = [b_1, b_2]$ are two intervals on R, then, the Hausdorff-metric between I_1 and I_2 is given by

$$d_H(I_1, I_2) = \max\{|a_1 - b_1|, |a_2 - b_2|\} \quad (43)$$

$$= \text{mid}I_1 - \text{mid}\, I_2| + |\text{spr}\, I_1 - \text{spr}\, I_2|$$

where $\text{mid}I_1 = \frac{a_1+a_2}{2}$, and $spr\ I_1 = \frac{a_2-a_1}{2}$.

The generalized Hausdorff-metric between fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as follows [45], [46]

$$D_p(\tilde{A},\tilde{B}) = \begin{cases} \left(\int_0^1 [d_H(A_\alpha, B_\alpha)]^p d\alpha\right)^{\frac{1}{p}} & p \geq 1 \\ \sup_{\alpha\in[0,1]} d_H(A_\alpha, B_\alpha) & p = 1 \end{cases} \quad (44)$$

Specially, the generalized Hausdorff-metric between symmetric triangular fuzzy numbers if $\widetilde{M} = (m, l_m)_T$ and $\widetilde{N} = (n, l_n)_T$ is as follows

$$D_1(\widetilde{M}, \widetilde{N}) = |m-n| + 0.5|l_m - l_n| \quad (45)$$

$$D_\infty(\widetilde{M}, \widetilde{N}) = |m-n| + |l_m - l_n| \quad (46)$$

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