



Theoretical Study on the Poisson's Ratio of Tubular Braid's Structure Considering Different Structural Deformation Regimes

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Article Information	Abstract
<p>Article history:</p> <p>Received: 2026-02-21</p> <p>Accepted: 2026-06-10</p>	<p>In the first part of this paper, an experimental study was conducted to investigate the effect of some important structural parameters of braid structure on Poisson's ratio. This paper aims to build some theoretical equations to predict the Poisson's ratio by geometrical modelling. Poisson's ratio of the braid structure was studied in three different parts. First, the Poisson's ratio was investigated at infinitesimal strains, and second, it was investigated at the area between two jamming of the structure (tensile and compressive jamming). Finally, an equation was derived to predict the Poisson's ratio of the braid structure at any point of its longitudinal strain. By using an exponential curve fitting, a modified equation was derived to predict the Poisson's ratio of the braid structure at an infinitesimal strain area. The results of verification were satisfying.</p>
<p>Keywords:</p> <p>Tubular Braid; Poisson's Ratio; Young's modulus; structural parameters.</p>	

1 INTRODUCTION

Braiding has long been used as an artistic method for the production of decorative textiles, but over time, especially in recent decades, the mechanical properties of the braid structure due to its complex, diverse and unique structure, attracted the attention of many experts in various fields.

Brunschweiler [1, 2] was the first to present his analysis of the braid structure. Using Peirce's hypotheses [3] in his analysis of the braid structure, he analyzed the geometry of a biaxial braid structure with a diamond-shaped repetitive unit cell. After that, many researchers studied the structure of the braid and the parameters affecting its mechanical properties [4-23]. Hristov et al. [8] studied theoretically and experimentally the mechanical properties of the tubular braid structure. Poisson's ratio of the braid structure and Poisson's ratio of its constituent yarns were among the parameters involved in the theoretical relations. In this study, to predict the maximum strength and maximum strain of the braid structure, the Poisson value of the braid structure and its constituent yarns were considered 0.5 and 0.25, respectively. In other words, in this paper, the volume of the braid structure was assumed to be constant during tension. Finally, considering the difference between the theoretical and experimental results, it was concluded that part of this

difference is due to the assumed values for Poisson's ratio of the braid structure.

Based on a review of previous studies, it was found that to date, there has been no comprehensive study on the deformational behavior of the braid structure and the effect of its structural parameters on Poisson's ratio. In this paper, we try to study the deformation behavior of the braid structure and its Poisson ratio using the method that is described below.

2 Theory

The tensile behaviour of the braid structure can be analysed in different parts of its stress-strain curve [25]. The angle between the braid axis and constituent strands is called the braid angle, which is the most important parameter that determines the mechanical properties of the braid structure and has a significant effect on the other structural parameters [2], so the focus of this study is to investigate the effect of braid angle on Poisson's ratio (PR) of braid structure and formula derivation. According to this point, in this part of the paper, the Poisson's ratio of the braid structure in three sections is examined theoretically as follows:

- Poisson's ratio of braid structures at infinitesimal strains

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- Poisson's ratio of braid structures in the area of geometrical changes (between compression jamming and tensile jamming)
- Poisson's ratio of braid structures in general (before and after geometrical changes).

2-1 Poisson's ratio of braid structure at infinitesimal strains

Figure 1 shows a diamond unit cell of a braid structure with its deformation. If it is assumed that the lengths of the sides forming the diamond unit cell (S) are constant during loading and deformation and only the braid angle (θ) is variable, then according to the mathematical definition of the instantaneous Poisson's ratio, following equations can be derived [24]:

$$x = 2S \sin(\theta) \quad (1)$$

$$y = 2S \cos(\theta) \quad (2)$$

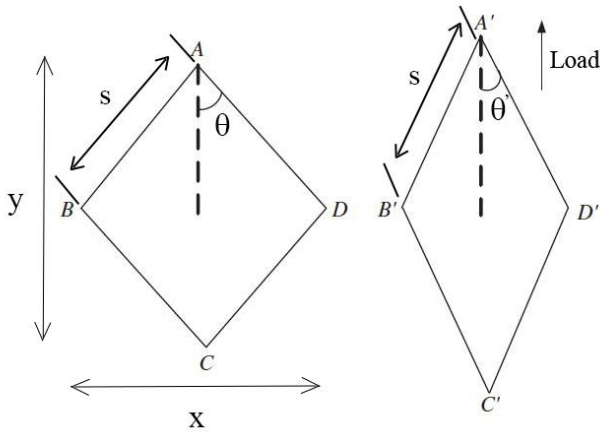


Figure 1 deformation of the braid's unit cell

Where (θ) is the braid angle before deformation and in its initial state, and θ' is the braid angle after deformation. Also, (x) and (y) are the width and length of the unit cell, respectively. Hence:

$$\frac{dx}{d\theta} = 2S \cos(\theta) \quad (3)$$

$$\frac{dy}{d\theta} = -2S \sin(\theta) \quad (4)$$

$$d\varepsilon_y = \frac{dy}{y} \quad (5)$$

$$d\varepsilon_x = \frac{dx}{x} \quad (6)$$

$$v_{yx}^{ins} = \frac{d\varepsilon_x}{d\varepsilon_y} = \frac{1}{\tan^2(\theta)} = \cot^2(\theta) \quad (7)$$

Where v_{yx}^{ins} is the instantaneous PR. Alternatively, if the structure of the braid changes by changing the length of the braid's threads (L) and the braid angle (θ) according to Figure 2, then the following equations could be derived:

$$L^2 = h^2 + r^2 \quad (8)$$

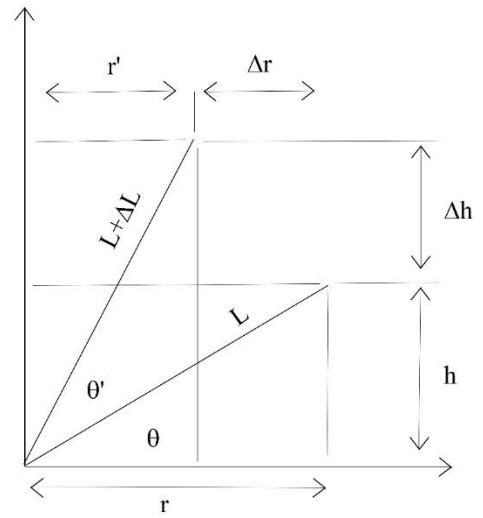


Figure 2 deformation of the braid structural

Where (h) and (r) are the height and radius of the tubular braid structure. Hence:

$$(L + \Delta L)^2 = (h + \Delta h)^2 + (r + \Delta r)^2 \quad (9)$$

$$\frac{\Delta L}{L} = \varepsilon_y \quad (10)$$

$$\frac{\Delta h}{h} = \varepsilon_b \quad (11)$$

$$\frac{\Delta r}{r} = \varepsilon_r \quad (12)$$

$$v_{yx}^{eng} = -\frac{\varepsilon_r}{\varepsilon_b} \quad (13)$$

Where v_{yx}^{eng} is the engineering PR. If it's assumed that the length of the unit cell is constant, then:

$$\Delta L = 0 \quad (14)$$

Given that this section examines the deformation of the braid structure at very low strain, it could be assumed:

$$\varepsilon_i^2 = 0 \quad (15)$$

$$1 = \cos^2(\theta) (1 + 2\varepsilon_b) + \sin^2(\theta) (1 - 2v_{yx}^{eng} \varepsilon_b) \quad (16)$$

$$\frac{\cos^2(\theta)}{\cos^2(\theta')} (1 + 2\varepsilon_b) = 1 \quad (17)$$

$$\varepsilon_b = \frac{\cos^2(\theta) - \cos^2(\theta')}{2 \cos^2(\theta)} \quad (18)$$

$$v_{yx}^{eng} = \cot^2(\theta) \quad (19)$$

As it turns out, Equation (19) equals Equation (7), which represents the equality of the engineering Poisson's ratio and the instantaneous Poisson's ratio at very low strains.

2-2 Poisson's ratio of braid structure in the area of geometrical changes (between compressive jamming and tensile jamming)

Figure 1 shows a diamond-shaped unit cell of a braid structure. If it is assumed that the length of the sides of the diamond unit cell remains constant during loading and

deformation and only the braid angle changes, then the following equations could be derived:

$$\varepsilon_r = \frac{2s \sin(\theta') - 2s \sin(\theta)}{2s \sin(\theta)} \quad (20)$$

$$\varepsilon_b = \frac{2s \cos(\theta') - 2s \cos(\theta)}{2s \cos(\theta)} \quad (21)$$

$$v_{yx}^{eng} = -\frac{\varepsilon_r}{\varepsilon_b} = -\left[\frac{\frac{\sin(\theta')}{\sin(\theta)} - 1}{\frac{\cos(\theta')}{\cos(\theta)} - 1} \right] \quad (22)$$

On the other hand, according to Figure 2 and the assumption that the length of the sides of the unit cell is constant and the braid angle is variable, a result similar to section 2.1 could be derived:

$$l^2 = h^2 + r^2 \quad (23)$$

$$(l + \Delta l)^2 = (h + \Delta h)^2 + (r + \Delta r)^2 \quad (24)$$

$$\frac{\Delta l}{l} = \varepsilon_y \quad (25)$$

$$\frac{\Delta h}{h} = \varepsilon_b \quad (26)$$

$$\frac{\Delta r}{r} = \varepsilon_r \quad (27)$$

$$v_{yx}^{eng} = -\frac{\varepsilon_r}{\varepsilon_b} \quad (28)$$

If it's assumed that the length of the unit cell is constant, $\Delta l = 0$ and by substitution of Equations (25-28) in Equation (24), then:

$$1 + \tan^2(\theta) = (1 + \varepsilon_b)^2 + \tan^2(\theta) (1 - v_{yx}^{eng} \varepsilon_b)^2 \quad (29)$$

$$\frac{\cos^2(\theta')}{\cos^2(\theta)} = (1 + \varepsilon_b)^2 \quad (30)$$

$$\varepsilon_b = -\left[\frac{\cos(\theta) + \cos(\theta')}{\cos(\theta)} \right] \quad (31)$$

$$v_{yx}^{eng} = -\frac{\varepsilon_r}{\varepsilon_b} = -\left[\frac{\frac{\sin(\theta')}{\sin(\theta)} - 1}{\frac{\cos(\theta')}{\cos(\theta)} - 1} \right] \quad (32)$$

As the comparison of relations (22) and (32) shows, these two relations are equal to each other, which confirms the chosen analytical method.

2-3 Poisson's ratio of braid structure in general (before and after geometric changes)

In general, according to Figure 2 and the contents mentioned in Section 2.2 and the assumption of increasing the length of the braid's yarns ($\Delta L \neq 0$) during loading, the following equations could be derived:

$$(1 + \varepsilon_y)^2 = \cos^2(\theta) (1 + \varepsilon_b)^2 + \sin^2(\theta) (1 - v_{yx}^{eng} \varepsilon_b)^2 \quad (33)$$

$$\varepsilon_y = \sqrt{\left[\cos^2(\theta) (1 + \varepsilon_b)^2 + \sin^2(\theta) (1 - v_{yx}^{eng} \varepsilon_b)^2 \right]} - 1 \quad (34)$$

$$\cos(\theta') = \frac{1 + \varepsilon_b}{\sqrt{(1 + \varepsilon_b)^2 + \tan^2(\theta) (1 - v_{yx}^{eng} \varepsilon_b)^2}} \quad (35)$$

$$v_{yx}^{eng} = \frac{\cos^2(\theta) \cos(\theta') - \cos(\theta') + \sqrt{\cos^2(\theta) (1 + \varepsilon_b)^2 (\cos^2(\theta') - 1) (\cos^2(\theta) - 1)}}{(\cos^2(\theta) - 1) \cos(\theta') \varepsilon_b} \quad (36)$$

3 Verification of theoretical relationships

Since a comprehensive experimental work on PR of tubular braids was the subject of the previous part of this paper, the experimental results reported in the literature [25] is used to verify the presented equations. Specifications of tubular braid samples produced with different structural parameters are given in Table 1. Sample's code comprised six sections, representing; braid angle, interlacement pattern, yarn count, braid's structural layers, braid's structural axis and the number of constituent braiding yarns, respectively.

In this section, the experimental and theoretical results of the initial Poisson's ratio are compared. It should be noted that the experimental data of the initial Poisson's ratio was measured by taking the slope of the first linear regions of true transversal strain - true longitudinal strain curves by piecewise curve fitting of samples as explained in the previous part of this paper [25]. Using the obtained equations (7) and (22), the instantaneous Poisson's ratio and engineering Poisson's ratio of braid structure with different braid angles are calculated and compared with experimental data, which were measured in the previous part of this paper, and the results are presented in Table 2 and Figure 3.

Table 1 Specifications of tubular braid samples

Sample	Braid angle (°)	Interlacement pattern	Yarn count (denier)	Structural layer	Structural axis	Yarn carriers
17-R-LSB32	17	Regular	1500	Single	Biaxial	32
27-R-LSB32	27	Regular	1500	Single	Biaxial	32
35-R-LSB32	35	Regular	1500	Single	Biaxial	32
40-R-LSB32	40	Regular	1500	Single	Biaxial	32
48-R-LSB32	48	Regular	1500	Single	Biaxial	32
17-T-LSB32	17	Twill	1500	Single	Biaxial	32
27-R-LDB32	27	Regular	1500	Double	Biaxial	32
27-R-HSB32	27	Regular	2500	Single	Biaxial	32
35-T-LSB32	35	Twill	1500	Single	Biaxial	32
40-R-HSB32	40	Regular	2500	Single	Biaxial	32
40-R-LDB32	40	Regular	1500	Double	Biaxial	32
40-R-HSB24	40	Regular	2500	Single	Biaxial	24
27-R-HSB24	27	Regular	2500	Single	Biaxial	24

Table 2 Experimental and theoretical PR

Sample	Instantaneous PR (Experimental)	Theoretical Instantaneous PR (Equation 7)	Theoretical Engineering PR (Equation 22)
17-R-LSB32	4.77	10.70	11.66
27-R-LSB32	3.18	3.85	4.00
35-R-LSB32	2.17	2.04	2.29
40-R-LSB32	1.30	1.42	1.57
48-R-LSB32	0.48	0.81	0.99

Table 3 Comparison of experimental and theoretical results

Sample	Experimental Results	Theoretical Results (Eq. 7)	Modified Theoretical Results (Eq. 37)	Error between experimental and theoretical results (%)
17-T-LSB32	4.37	10.70	4.84	9.71
17-R-LSB32	4.77	10.70	4.84	1.45
27-R-LDB32	2.87	3.85	3.11	7.72
27-R-LSB32	3.18	3.85	3.11	-2.25
27-R-HSB32	1.91	3.85	3.11	38.58
35-T-LSB32	1.43	2.04	2.04	29.90
35-R-LSB32	2.17	2.04	2.04	-6.37
40-R-HSB32	1.17	1.42	1.43	18.18
40-R-LDB32	0.74	1.42	1.43	48.25
40-R-LSB32	1.30	1.42	1.43	9.09
48-R-LSB32	0.48	0.81	0.48	0
40-R-HSB24	1.17	1.42	1.43	18.18
27-R-HSB24	3.48	3.85	3.11	-11.90

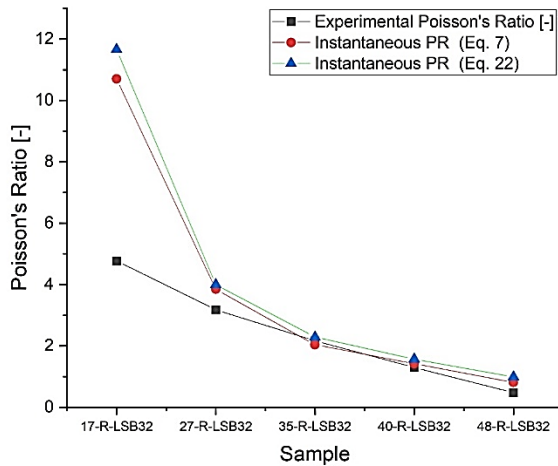


Figure 3 Comparison of theoretical and experimental results of the initial Poisson's ratio

Considering that equation (22) was extracted based on the geometrical changes of the braid structure (between compressive and tensile jamming) and the geometrical changes of the braid structure also occur in the small strains, Figure 3 showed that at low strains, the theoretical results of the instantaneous Poisson's ratio and the engineering Poisson's ratio are very close to each other. Because the theoretical results of the instantaneous Poisson's ratio and the engineering Poisson's ratio are very close to each other, the following equation (7) is used as a theoretical relation for verification.

Figure 4 was obtained by plotting theoretical data (Equation (7)) and experimental instantaneous Poisson's ratio. As shown in Figure 3 and Figure 4, the difference between the theoretical and experimental data at the lower braid angles is

very high, and decreases with increasing braid angle. Then, by using an exponential regression curve fitting shown in Figure 4, a high coefficient of determination was established between the mentioned curve and the fitted curve.

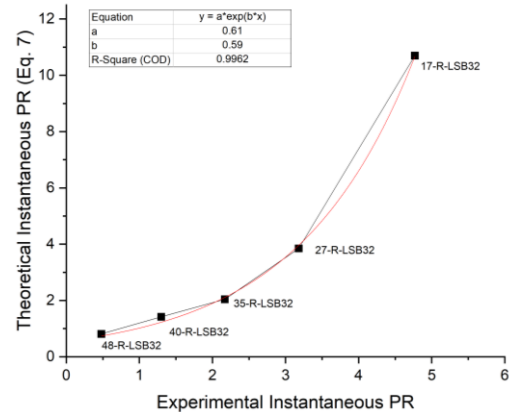


Figure 4 Relationship between theoretical and experimental results of instantaneous Poisson's ratio

This point is important because it can be understood that a factor with an exponential effect affects the Poisson's ratio that has been neglected in theoretical relations. As mentioned in the previous part of this paper, one of the common and effective factors on the Poisson's ratio among all samples was the amount of yarn's crimp and take up of the braid structure, followed by the stiffness of the braid structure, compression, and friction between the braid yarns. Considering the comparison of theoretical and experimental results, it can be said that the effect of the mentioned factors is probably exponential, so that this difference is high in low braid angles and decreases in high braid angles.

According to the contents of the previous part of this paper, where one of the key findings was that the dominant effect of braid angle over other parameters, and according to the points mentioned in this section, to apply a correction coefficient in equation (7), the mentioned exponential fit was used and the modified equation to calculate the initial Poisson's ratio was introduced as follows:

$$v_{\text{mod}}^{\text{ins}} = 1.69 \times \ln(1.64 \times v_{\text{yx}}^{\text{ins}}) \quad (37)$$

According to Figure 5 and Table 3, a good relationship was established between the experimental and theoretical results of the initial Poisson's ratio. It is important to note that, as in the previous part of this paper, the focus of the experimental works was on the biaxial braids; in comparing the theoretical and experimental results, only biaxial samples were examined [25]. The biaxial braid is characterized by two bias yarn systems, which are interwoven at an arbitrary braiding angle. The triaxial braid is characterized by a third yarn system, the zero-degree yarns, which are positioned in the braid in parallel to the mandrel axis, and the study of theoretical relations would be complicated and out of the scope of this paper [14].

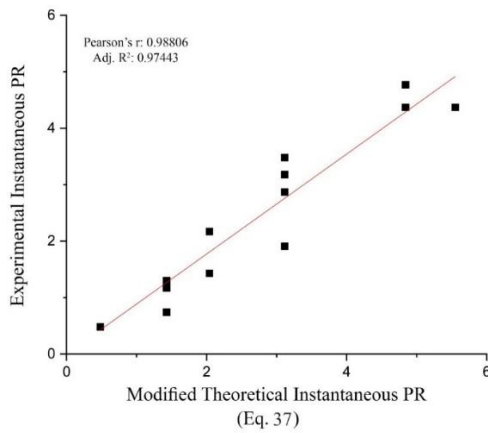


Figure 5 Relationship between modified experimental and theoretical results

To investigate and verify the relation (36), sample 35-R-LSB32 was selected as an example, and the amount of braid angle was measured in each percent of the strain of the braid structure. After entering the variable values in equation (36), Figure 6 was obtained. As can be seen from Figure 6, the difference between the theoretical and experimental values of the engineering Poisson's ratio in the first region is very high, which decreases with increasing strain of the braid structure.

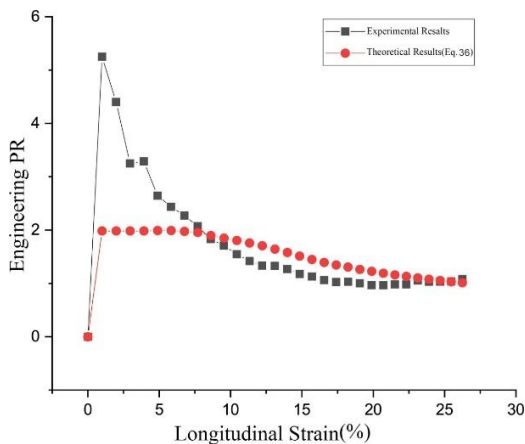


Figure 6 Comparison of theoretical and experimental results in general

Considering the contents mentioned in previous sections, regarding the similarity of the values of the engineering and instantaneous Poisson's ratio at very low strains and Figure 6, it can be concluded that according to the complex mechanical behavior of the braid structure and the constituent fibers, to estimate the value of the initial Poisson's ratio, the modified equation (37) with acceptable accuracy, and for high strains, the equation (36) can be very useful.

4 Conclusion

This paper aims to build some theoretical equations to predict the Poisson's ratio by geometrical modelling. Poisson's ratio of the braid structure was studied in three different parts. First, the Poisson's ratio was investigated at infinitesimal strains, and second, it was investigated at the area between two jamming of the structure (tensile and compressive jamming). Finally, an equation was derived to predict the Poisson's ratio of the braid structure at any point of its longitudinal strain. By using an exponential curve fitting, a modified equation was derived to predict the Poisson's ratio of the braid structure at an infinitesimal strain area. It could be concluded that for estimating the Poisson's ratio in high strains and low strains, equations 36 and 37 could be useful, respectively.

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