An Analytical Study of Initial Shear Behavior of Plain Woven Hybrid Fabrics

Majid Tehrani-Dehkordi and Hooshang Nosraty

***Abstract*—During recent years, the fabrics made from different yarns have been used for industrial purposes such as composite materials. Given the importance of the shear properties of these fabrics, a mechanical model based on the Grosberg and Park's Model for hybrid plain-woven fabrics is proposed. In this model, using the Energy method, the initial load-shear angle behavior of hybrid fabrics is calculated from the yarn mechanical properties and the fabric structure. To evaluate the proposed model, the experimental results for initial shear modulus obtained from pure and hybrid fabrics of basalt and nylon are compared with the theoretical model results. A comparison of the results shows that there is rather a good agreement between the proposed model predictions and experimental results.**

***Keywords*: energy method, hybrid, initial modulus, shear behavior, woven fabric**

I. Introduction

DUring recent years, the use of composites reinforced with woven fabrics has increased. This type of composites due to their properties such as high strength to weight ratio, low vibration and high corrosion resistance have gradually replaced the metals in different industries e.g. car and air-plane industries [1,2]. In spite of the above-mentioned advantages, polymer composites due to some problems in their production process in three- dimensional space usually cannot be produced by automatic machines [1]. In this type of composites, two dimensional woven fabrics should be shaped properly as reinforcing layers over the three-dimensional spaces such as spherical parts. When the two dimensional fabrics are put on the spherical parts, in order to prevent tearing and wrinkling of the fabric, the tensile forces are implemented on the fabric in different directions. Imposed forces on the fabric which are not in the warp and weft directions create shear forces in the fabric structure. The created shear forces cause slippage of yarns, and changes in angle and surface density in different regions of a spherical part. This eventually will lead to unequal resin permeability and different mechanical properties in various parts of a spherical body [3-5]. In order to control the variation of angles between the yarns in protuberant parts and to improve the performance of composites in three- dimensional space, in most cases, these types of composites are produced by hand, since they cannot be produced in an automatic process. Therefore, production cost and final price of these composite parts will noticeably increase. To mechanize the production process of composites used in three-dimensional spaces, many researchers have studied the forming of the fabrics on spherical regions and their shear properties during the forming process. In these researches the formability and shear properties of woven fabrics with respect to their yarn properties and geometrical parameters have been studied using various methods. Sun and Pan [5], Robertson [6], Suemasu [7], Taylor and Mack [8], Lindberg *et al.* [9], Grosberg and Park [10], Kawabata *et al.* [11], Sinomery and Drean [12] and Skelton [13] in separate studies presented analytical equations for the relation between shear force and shear angle. Hofstee and Keulen [2], Yu and Daghyani [4], Martin [14], Ya [15], Page [16], Boisse [17] and Cao [18] studied the formability and shear properties of woven fabrics by numerical methods.

Nowadays, hybrid fabrics made from different types of yarns are produced for use in composite industries [19,20]. In these types of fabrics, yarns with different properties in warp, weft, or both directions are placed with a certain regulation. In reviewing the literature no research was found on shear properties of woven hybrid fabrics. Regarding the importance of the formability and shear properties of these fabrics in composite and clothing industries, in this paper the Grosberg and Park's model [10] has been developed for hybrid fabrics.

II. Analytical Model

In this part, based on the Grosberg and Park's model [10], an analytical model for hybrid fabrics with plain weave at low shear angle is presented.

A. Assumptions and Calculation Basis

Grosberg and Park [10] demonstrated that the modes of deformation involve several forms depending upon the degrees of shear imposed on the fabric. These are:

a) Deformation due to rigid intersections when the shear force is too small to overcome the friction;
b) Yarn slippage at the intersection. This only takes place when the shear force overcomes the friction;
c) An elastic deformation when slipping is complete;
d) Jamming in the structure.

The analysis method and the parameters affecting the shear process for each of the above phases are different. By considering certain assumptions for each shear phase researchers have investigated the shear properties in that

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area. Grosberg and Park [10], by using the energy method, obtained the shear properties of plain woven fabrics from the mechanical properties of yarns and the geometrical parameters of fabrics. They studied the shear process at low forces (the first phase of shear behavior) using proper assumptions. Their assumptions were as follows:

a) According to Fig. 1(a), the fabric is like a network in which the warp and weft yarns are connected together as a joint in contact points.
b) The shear force only causes the rotation of yarns on each other and they don't slip in their contact points.
c) The yarns are inextensible and incompressible.
d) The deformation of yarns only is due to their bending deformation.
e) The size of yarns' contact area in both warp and weft directions is equal.

In addition to the above assumptions, Sun and Pan [5] found that according to Fig. 1(b), at the beginning of the shear process, the employed force on each yarn is only shear force (*F*) but by changing the angle between the yarns this force is converted to two forces: shear force (*f*) and tensile force (*t*). Since in this study the shear properties have been investigated on low angle variations, the created tensile force is very small and negligible.

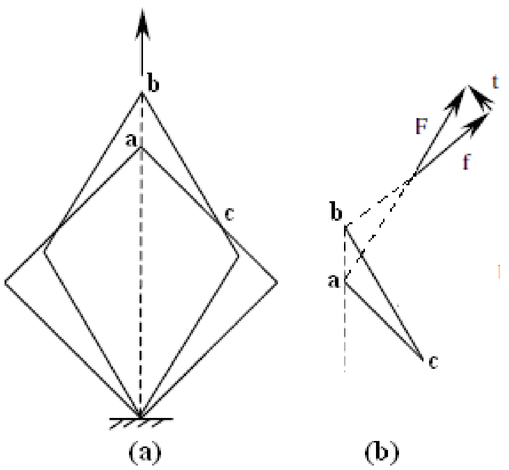

Fig. 1. Surface shear on the fabric due to the low applied shear force.

Given the above-mentioned assumptions, Grosberg and Park presented (1) for the relation between the shear force and shear angle and (2) for calculating the shear modulus.

$$\theta = \frac{F}{12BL}\left[\frac{P_1}{P_2}(l_2 - d)^3 + \frac{P_2}{P_1}(l_1 - d)^3\right] \quad (1)$$

$$G = F/\theta \Rightarrow G = 12BL\left[\frac{P_1}{P_2}(l_2 - d)^3 + \frac{P_2}{P_1}(l_1 - d)^3\right]^{-1} \quad (2)$$

In these equations, θ and F are the shear angle and the shear force, respectively; G is the shear modulus; B is the bending stiffness of the yarns, L is the length of the sample, l and P are the length and distance between the yarns respectively (according to Fig. 2), and d is the contact length of warp and weft yarns. The subscripts 1 and 2 are related to warp and weft directions, respectively.

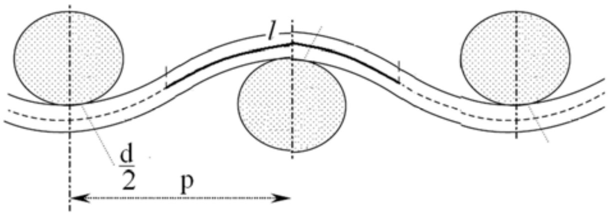

Fig. 2. Geometrical parameters of the Grosberg and Park theoretical model.

B. Surface Shear Properties of Hybrid Fabrics

Fig. 3(a) shows a view of a hybrid woven fabric. As the figure shows, the yarn properties have changed in warp and weft directions at over two repeat values in unit cell of the fabric. For ease of calculation, the fabric is divided into four parts. Each part has different yarns. Given the Grosberg and Park's model's assumptions and assuming an equal distance between different yarns centers we can rewrite (1) for each part of the fabric unit cell as follows:

$$F_i = 12B_i^* L\theta_i\left[\frac{P_1}{P_2}(l_{ik} - d_i)^3 + \frac{P_2}{P_1}(l_{ij} - d_i)^3\right]^{-1} \quad (3)$$

In this equation and other equations in this section, the subscript i shows the sections of Fig. 3(a) and the subscripts j and k represent the related yarns for that section. These subscripts are presented in Table I.

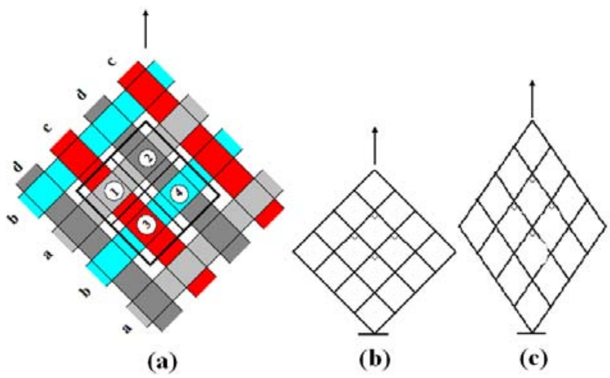

Fig. 3. The sample of hybrid fabric under surface shear force.

TABLE I
PRESENTATION OF THE EQUATION SUBSCRIPTS RELATED TO FIG. 3(A)

Section No.	*i*	*j*	*k*
1	1	a	c
2	2	a	d
3	3	b	c
4	4	b	d

In (3), B_i^* is the equivalent bending stiffness of yarns in each section. This parameter for each section of fabric is calculated based on (4). This equation is proposed by Sinomeri and Drean for unequal yarns [10].

$$B_i^* = 2B_j B_k / (B_j + B_k) \quad (4)$$

Other parameters in (3) are the same as in (1).

As mentioned before, the fabric is assumed as a network in which the yarns are connected like joints and they can only rotate on each other. Therefore, regarding Fig. 3(b) and (c) the angle between the warp and weft yarns and hence the shear angle for all of the sections at each time of the shear process is the same.

$$\theta = \theta_1 = \theta_2 = \theta_3 = \theta_4 \quad (5)$$

If the shear force at each contact point is assumed as the mean value of the shear force at all points, it can be concluded that:

$$F = (F_1 + F_2 + F_3 + F_4)/4 \quad (6)$$

By replacing (3) in (6) and considering (5), the following equation is concluded.

$$F = \left(\theta \sum_{i=1}^{4} \left[12B_i^* L \left(\frac{P_1}{P_2}(l_{ik} - d_i)^3 + \frac{P_2}{P_1}(l_{ij} - d_i)^3 \right)^{-1} \right] \right) \Big/ 4 \quad (7)$$

Equation (7) shows the relation between the shear force and shear angle for the hybrid woven fabric presented in Fig. 3(a).

Regarding (2) and (7) the shear modulus for the hybrid fabric is written as follows:

$$G = \sum_{i=1}^{4} \left[12B_i^* L \left(\frac{P_1}{P_2}(l_{ik} - d_i)^3 + \frac{P_2}{P_1}(l_{ij} - d_i)^3 \right)^{-1} \right] \Big/ 4 \quad (8)$$

In this equation, G is determined in N.Rad^{-1}. As shown in (7) and (8) the shear properties of the hybrid woven fabric depends on the yarn mechanical properties and fabric geometrical parameters. One can study the shear properties of hybrid woven fabrics with other combinations in the same manner.

III. Experimental

To evaluate the proposed equations, the theoretical and experimental results of some hybrid fabrics composed of different arrangements of basalt and nylon yarns are compared. The physical and mechanical properties of basalt and nylon yarns are given in Table II.

TABLE II
PHYSICAL AND MECHANICAL PROPERTIES OF FIBERS AND MATRIX

	Density (kg/m^{-3})	Tensile modulus (GPa)	Tensile strength (MPa)	Strain at break (%)
Basalt	2700	85	1800	2
Nylon 6	1250	2.45	1000	20.5

An Iwer rigid single rapier loom is used to weave the fabrics. Fig. 4 shows a view of the yarn arrangement in the fabric samples. In this figure, the fabrics are coded based on their composed yarn material (basalt (B) or nylon (N)).

The required geometrical parameters for theoretical equations are obtained by providing a cross-section of the fabric, image processing and microscopic methods. Figs. 5(a) and (b) show a cross section area of basalt and nylon yarns, respectively. The bending properties of the yarns were measured using a pure bending test device. This device measures the curve variations against implemented bending momentum variations. The fabric geometrical parameters and the yarn mechanical properties are summarized in Table III.

In this table, l, d and P are determined in cm and B is determined in N.cm^2.

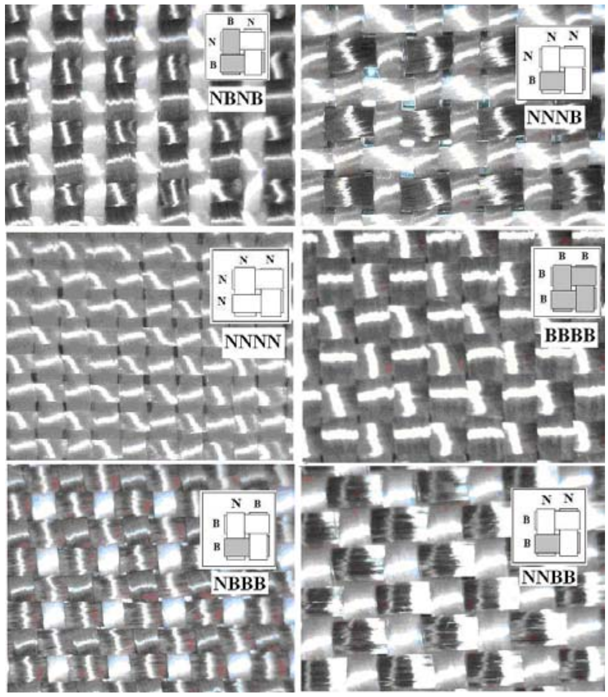

Fig. 4. A view of the investigated fabric samples.

Fig. 5. Typical cross section areas of fabrics, (a) basalt yarn, (b) nylon yarn.

The theoretical shear properties of investigated fabric samples are calculated using the data in Table III and equations (7) and (8). The experimental shear properties of fabric samples are measured by implementing the load along 45° direction in fabrics [21] and using 5566 Instron Tensile Tester device. This device is equipped with a 10 kN load cell. Fig. 6 shows the shear loading device (a), prepared sample for shear testing (b), and the fabric sample during shear loading (c).

TABLE III
GEOMETRICAL PARAMETERS OF FABRICS AND MECHANICAL PROPERTIES OF YARNS

Fabric code	*NNNN*	*NNNB*	*NNBB*	*NBNB*	*NBBB*	*BBBB*
l_{1a}	0.214	0.210	0.212	0.210	0.201	0.206
l_{1c}	0.211	0.217	0.214	0.216	0.220	0.207
l_{2a}	0.214	0.207	0.213	0.210	0.201	0.206
l_{2d}	0.211	0.216	0.213	0.216	0.220	0.207
l_{3b}	0.214	0.210	0.213	0.210	0.205	0.206
l_{3c}	0.211	0.217	0.216	0.216	0.219	0.207
l_{4b}	0.214	0.207	0.216	0.210	0.205	0.206
l_{4d}	0.211	0.216	0.216	0.216	0.219	0.207
B_a	0.048	0.048	0.048	0.048	0.048	0.106
B_b	0.048	0.048	0.106	0.048	0.106	0.106
B_c	0.048	0.048	0.048	0.106	0.106	0.106
B_d	0.048	0.106	0.106	0.106	0.106	0.106
d_1	0.088	0.080	0.069	0.080	0.082	0.084
d_2	0.089	0.081	0.069	0.072	0.085	0.086
d_3	0.087	0.081	0.092	0.074	0.080	0.087
d_4	0.088	0.084	0.083	0.079	0.078	0.086
P_1	0.201	0.210	0.207	0.209	0.211	0.200
P_2	0.203	0.196	0.205	0.198	0.193	0.201

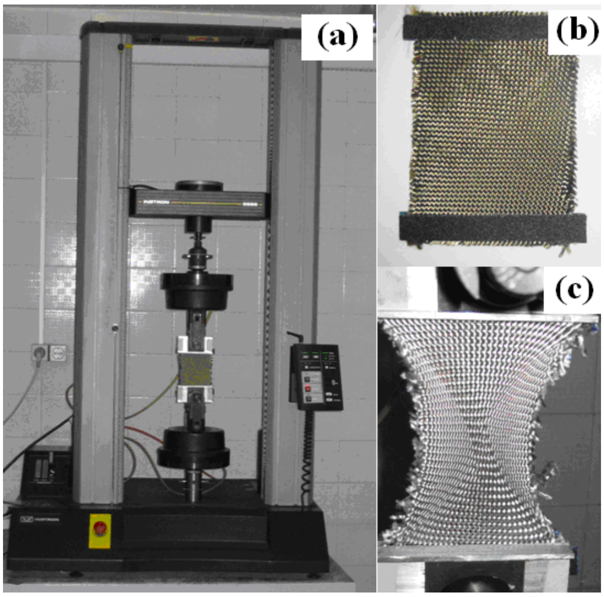

Fig. 6. (a) shear loading device, (b) prepared sample for shear test, (c) fabric sample during shear loading.

IV. RESULTS AND DISCUSSION

Fig. 7 shows the theoretical and experimental shear force with respect to the shear angle for different hybrid woven fabrics at low shear angle. In this figure, the shear angle at any point of the shear process is calculated using the following equation:

$$\theta = 90 - (360/\pi) Arc\cos\left(1/\sqrt{2} + \delta/2L\right) \quad (9)$$

where δ is the displacement of the crosshead and L is the side length of the picture frame rig [22].

Although the determination of shear forces at low shear angles does not show complete information about the fabric shear behavior, investigating the shear process at this range can determine valuable data such as shear modulus. The theoretical and experimental results for initial shear modulus are summarized in Table IV.

TABLE IV
THEORETICAL AND EXPERIMENTAL SHEAR MODULES OF DIFFERENT HYBRID FABRICS

Fabric code	Shear modulus (N/Degree)		Difference (%)
	Theory	Experiment	
NNNN	15.6	10.2	53
NNNB	15.8	11.1	42
NBNB	17.7	12.1	46
NNBB	17.2	11.6	48
NBBB	23.6	15.9	48
BBBB	38.5	28.1	37

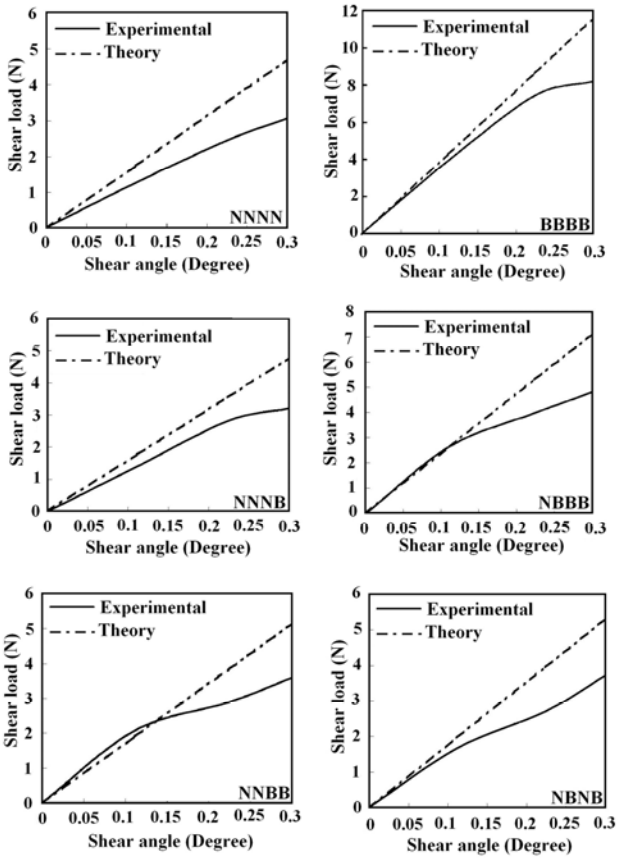

Fig. 7. Variation of theoretical and experimental shear force with shear angle for different hybrid woven fabrics.

In this table, the experimental shear modulus is obtained from the slope of the shear force – shear angle diagram in the range of zero to 0.3 degree. In other words, for experimental shear modulus the slope of the shear force – shear angle diagram in its initial region was considered in which the imposed shear force causes the rotation of yarns at connection points; hence the first type of shear happens. In this region, the slippage of yarns over each other is negligible. The results presented in Table IV show that the difference between the theoretical and experimental shear modulus varies from 37 to 53 percents for pure and hybrid fabric samples. The percentage of difference is the same as the results obtained by previous researches who investigated the first phase of shear behavior [3,5,9]. By comparing theoretical and experimental results in Table IV, it can be found that the pure nylon fabric (NNNN) has the most difference. The applied model for pure fabrics at low shear angle is approved by former investigation [3,9,10]. Thus it seems that the differences between the theoretical and experimental shear modulus are not related to the development of the Grosberg and Park model for hybrid woven fabrics. These differences can be due to the mistakes in the measurement of experimental bending stiffness of the yarns, the experimental shear tests, the preparation of fabric cross section, ignoring the yarn slippage in fabric and also ignoring the tensile force in the theoretical model. One of the error sources which have a considerable effect on the theoretical results is the determination of yarns contact length (d). In this case regarding the dependence of contact length to the cross section shape and diameter of the yarns, the probability of error would be high [3,16].

In Fig. 7, the trend of the variation of the theoretical and experimental shear forces with shear angle is shown. As it can be observed from shear force – shear angle curves, the theoretical and experimental results have a good agreement in the initial region, to an extent that in some of the samples the theoretical and experimental results overlap. By increasing the shear angle, the theoretical shear force becomes more than the experimental shear force gradually. This diversion can result from ignoring the tensile force due to shear angle and the slight slippage of yarns [21]. The trend of the theoretical and experimental shear force variation in this work is the same as the results obtained by Nguyen *et al*. [3] and Sun and Pan [5] for pure carbon fabrics in previous studies.

Regarding the cited subjects in this section, we can conclude that the developed model for shear behavior of hybrid fabrics at low shear angle predicts the shear behavior of these types of fabrics fairly good. For improvement of theoretical results, the parameters of yarn slippage and yarn extension during the shear process should be considered in the developed model.

V. Conclusion

Shear behavior of plain woven fabrics has been investigated by some researchers using different methods. In this research an analytical equation based on the Grosberg and Park's model was developed for studying the hybrid plain woven fabrics by considering their applications in different industries. For evaluation of obtained equations, the theoretical and experimental results of basalt – nylon hybrid woven fabrics were compared. The results indicated that:

- The difference between the theoretical and experimental shear modulus for different samples ranged from 37 to 53 percents.
- The comparison of the pure and hybrid fabrics revealed that the highest difference between the theoretical and experimental results of shear modulus belonged to the pure fabric. Thus the difference between the theoretical and experimental results was not only related to the presented model, and it could be due to other error sources.
- The most important error source which had a considerable effect on the theoretical results was the determination of yarns contact length.
- The trend of the variation of the theoretical and experimental shear forces with shear angle for different fabrics had fairly good agreement.
- For the improvement of the theoretical results, the slippage and extension of yarns during shear process should be considered in the developed model.

Acknowledgements

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